**Integration – 2023 June A2 Math 9709**

**1.** June/2023/Paper_9709/31/No.8

Let $f(x) = \dfrac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

**(a)** Express $f(x)$ in partial fractions. [5]

**(b)** Hence find the exact value of $\int_0^4 f(x)\,dx$, giving your answer in the form $a + b \ln c$, where $a$, $b$ and $c$ are integers. [5]

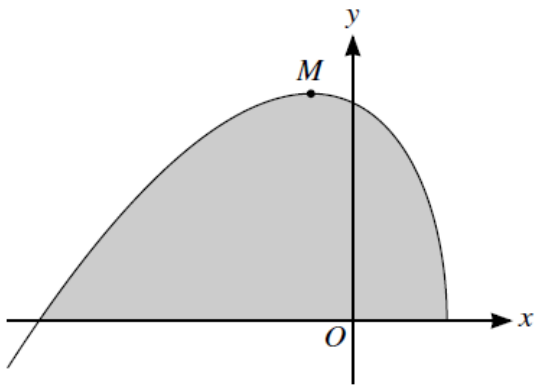
**2.** June/2023/Paper_9709/32/No.9

Let $f(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

**(a)** Express $f(x)$ in partial fractions. [5]

**(b)** Hence show that $\int_0^1 f(x)\,dx = \frac{5}{2} - \ln 72$. [5]

**3.** June/2023/Paper_9709/32/No.10(b)




The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point $M$.

**(b)** Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the $x$-axis. Give your answer in the form $a\sqrt{13}$, where $a$ is a rational number. [5]

**4.** June/2023/Paper_9709/33/No.7

**(a)** Use the substitution $u = \cos x$ to show that

$$\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx = \int_{-1}^{1} 2u e^{2u} \, du. \qquad [4]$$

**(b)** Hence find the exact value of $\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx$. [4]