Algebra – 2023 Nov CIE Mathematics

1. Nov/2023/Paper_9709/21/No.4

(a) Sketch, on the same diagram, the graphs of $y = |3x - 5|$ and $y = 2x + 7$.   [2]

(b) Solve the equation $|3x - 5| = 2x + 7$.   [3]

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(c) Hence solve the equation $|3^{y+1} - 5| = 2 \times 3^y + 7$, giving your answer correct to 3 significant figures.   [2]
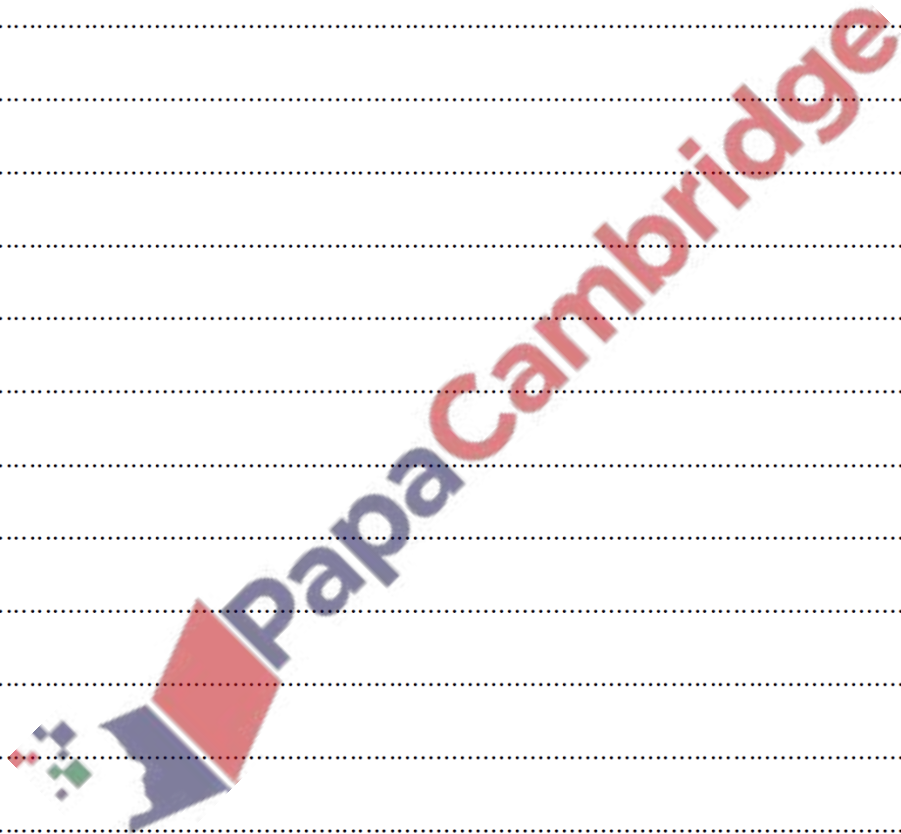
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The polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + ax^2 + bx - 20,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is -11 when $p(x)$ is divided by $(x + 1)$.

(a) Find the values of a and b. [5]

(b) Hence factorise p(x), and determine the exact roots of the equation p($3x$) = 0. [4]

When the polynomial

$$ax^3 + 4ax^2 - 7x - 5$$

is divided by $(x + 2)$, the remainder is 33.

Find the value of the constant a. [2]

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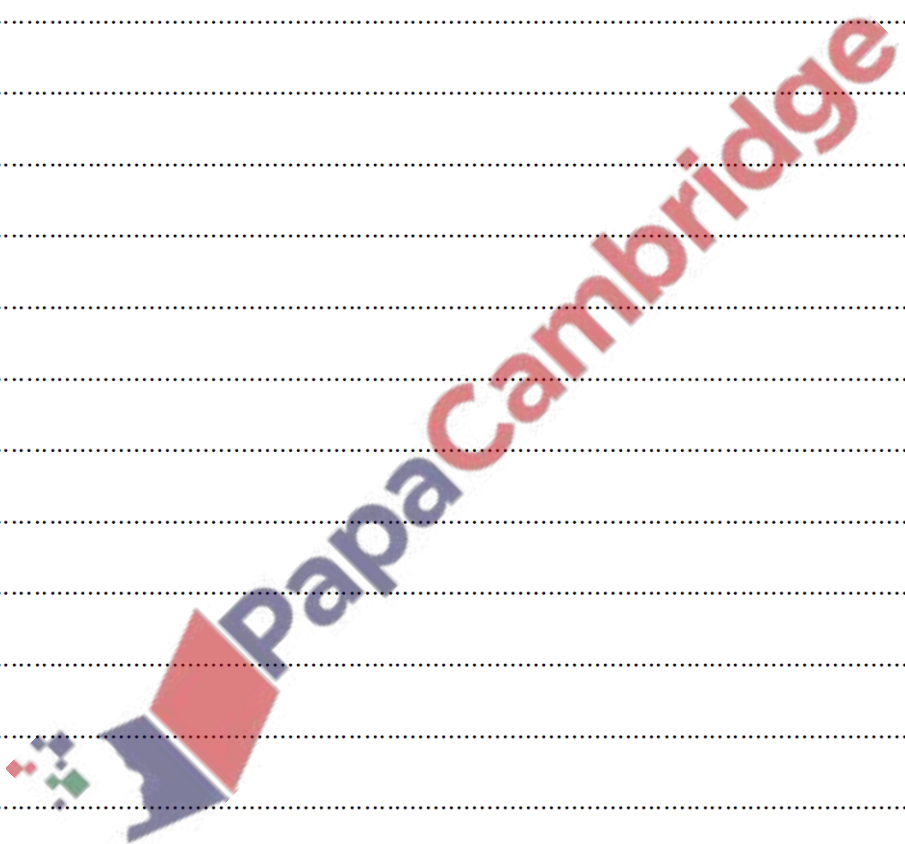..

(a) Sketch, on the same diagram, the graphs of $y = |3 - x|$ and $y = 9 - 2x$. [2]

(b) Solve the inequality $|3 - x| > 9 - 2x$. [3]

(c) Use logarithms to solve the inequality $2^{3x-10} < 500$. Give your answer in the form $x < a$, where the value of a is given correct to 3 significant figures. [3]

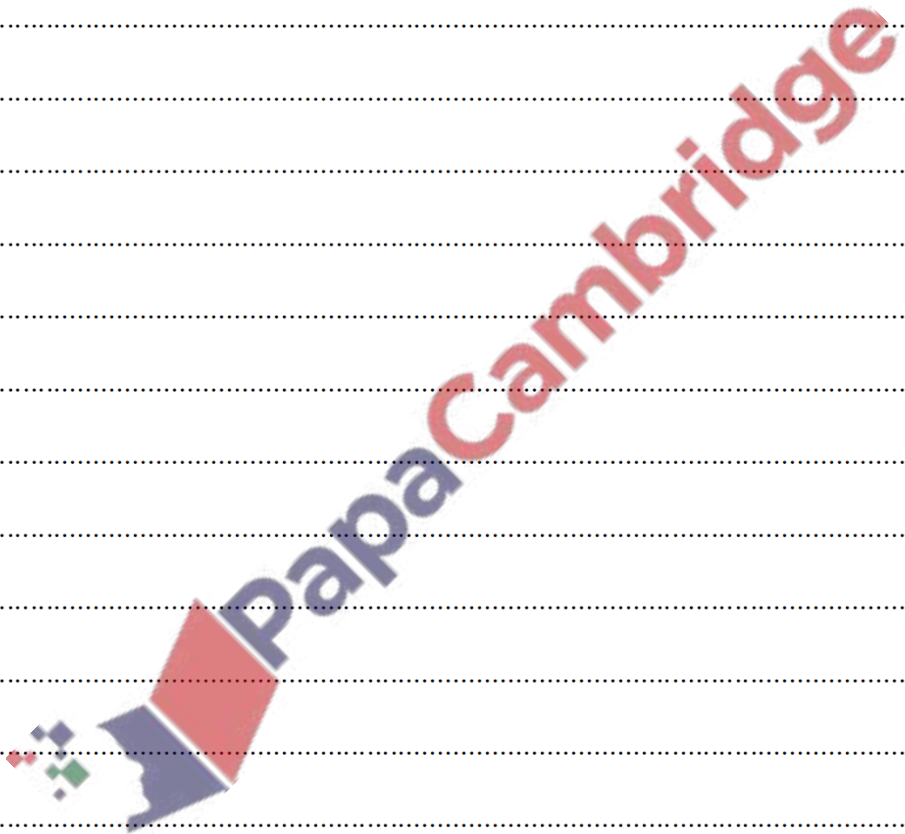(d) List the integers that satisfy both of the inequalities $|3 - x| > 9 - 2x$ and $2^{3x-10} < 500$. [1]

5.

Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2.

[5]

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(c) State the set of values of x for which the expansion in **(b)** is valid.

[1]

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6.

(a) Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

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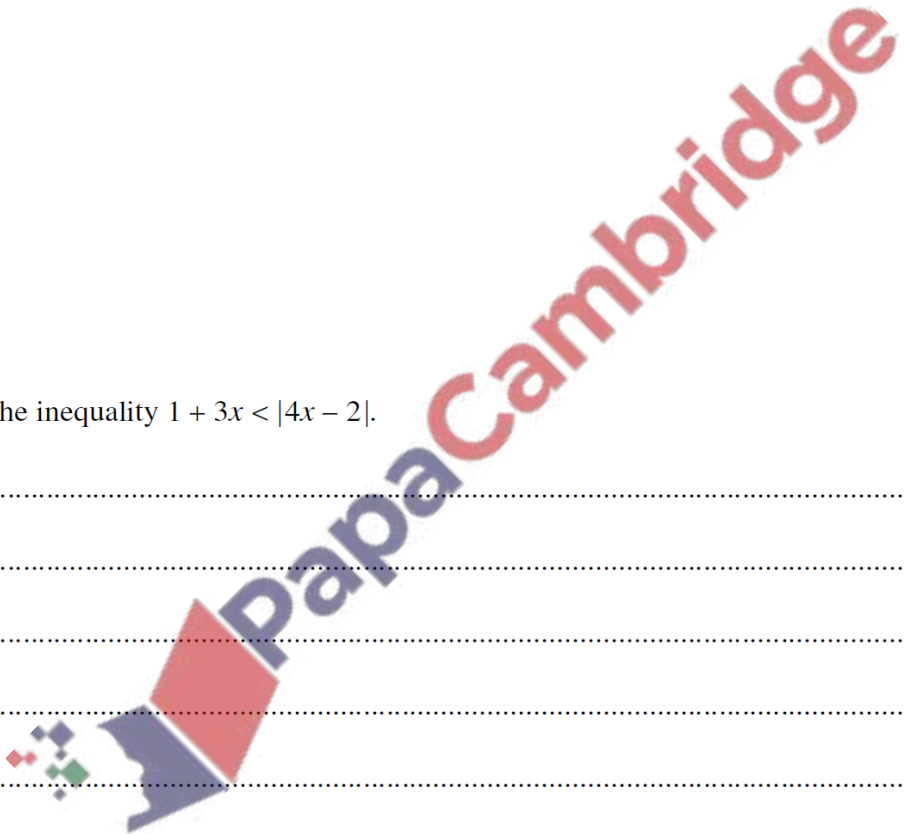
The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by p(x). It is given that p(x) is divisible by $(2x - 1)$ and that when p(x) is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

8.

Find the set of values of x satisfying the inequality $\left|2^{x+1} - 2\right| < 0.5$, giving your answer to 3 significant figures. [4]
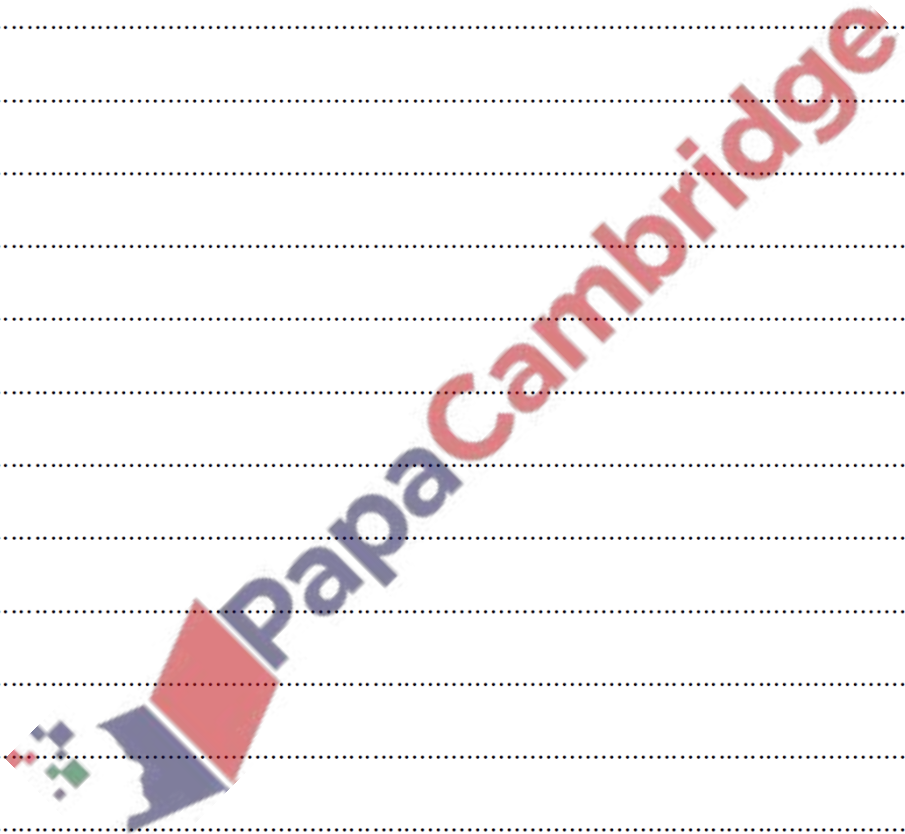
The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $(x + 2)$ the remainder is -38 and when $p(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b. [5]

Let $f(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^3.
[5]

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(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form.
[1]

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