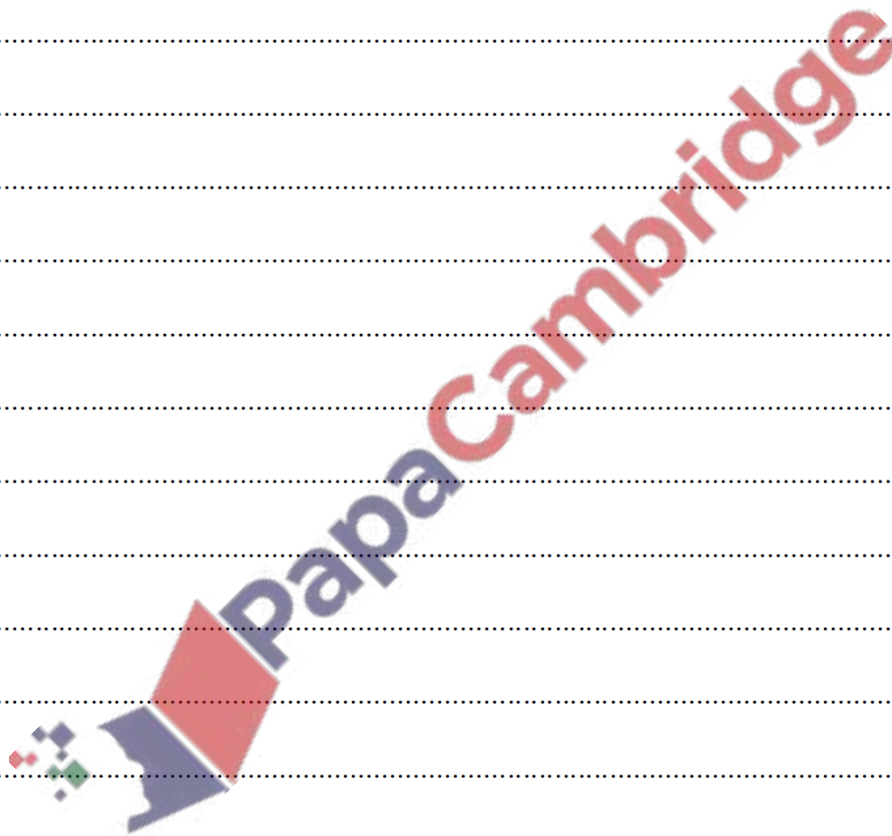
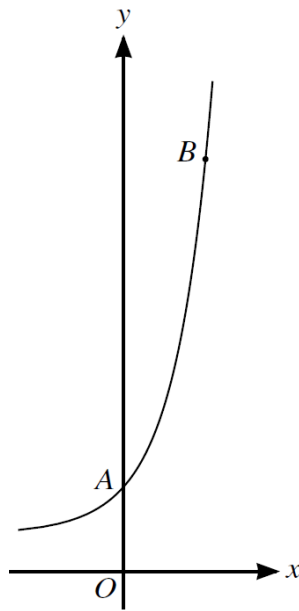


1. **Nov/2023/Paper_9709/21/No.2**

A curve has equation $y = 3 \tan \frac{1}{2}x \cos 2x$.

Find the gradient of the curve at the point for which $x = \frac{1}{3}\pi$. [5]





The diagram shows the curve with parametric equations

$$x = 3 \ln(2t - 3), \quad y = 4t \ln t.$$

The curve crosses the y-axis at the point A. At the point B, the gradient of the curve is 12.

- (a) Find the exact gradient of the curve at A.

[5]

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(b) Show that the value of the parameter t at B satisfies the equation

$$t = \frac{9}{1 + \ln t} + \frac{3}{2}. \quad [2]$$

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(c) Use an iterative formula, based on the equation in (b), to find the value of t at B , giving your answer correct to 3 significant figures. Use an initial value of 5 and give the result of each iteration to 5 significant figures. [3]

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Find the exact coordinates of the points on the curve $y = \frac{x^2}{1 - 3x}$ at which the gradient of the tangent is equal to 8. [5]

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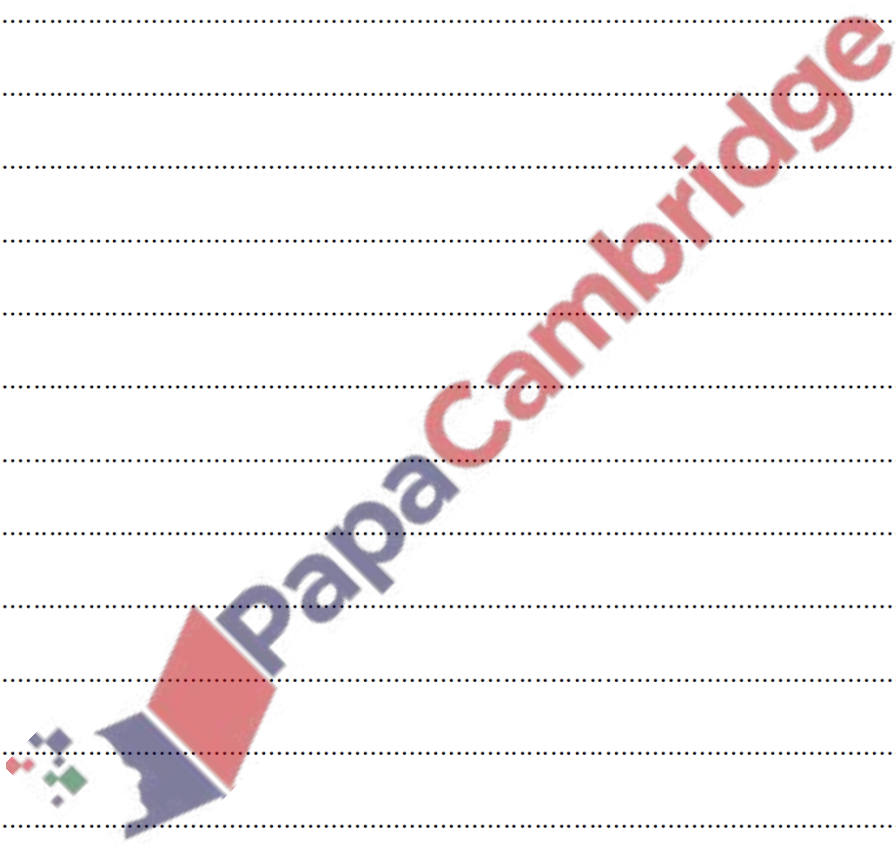
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The parametric equations of a curve are

$$x = \sqrt{t} + 3, \quad y = \ln t,$$

for $t > 0$.

- (a) Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t . [3]

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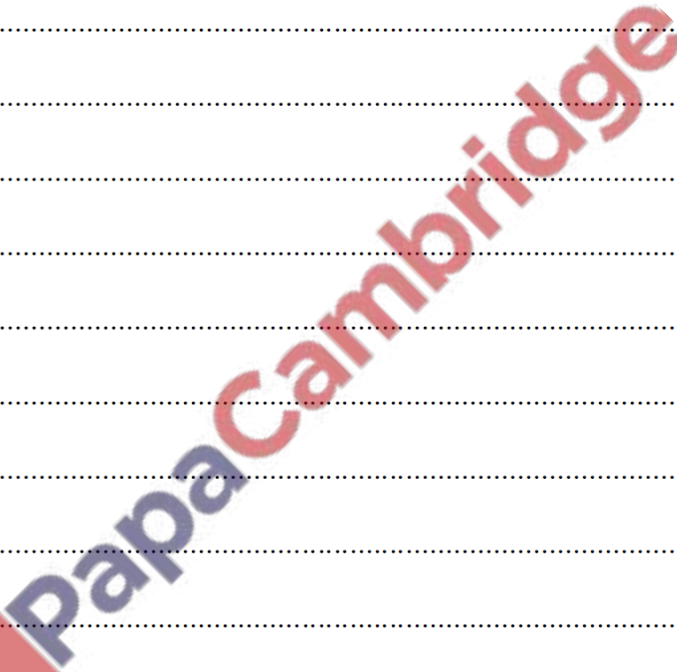
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- (b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2 . [3]

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The parametric equations of a curve are

$$x = (\ln t)^2, \quad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer.

[4]

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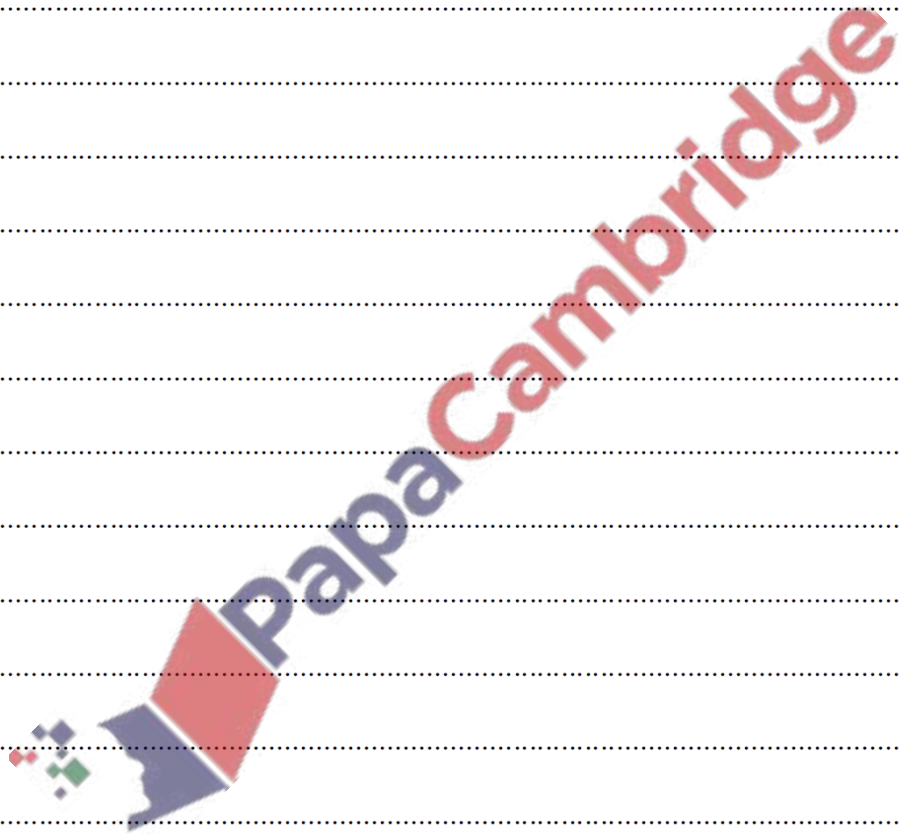
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(b) Hence find the coordinates of the points on the curve at which the tangent is parallel to the x -axis. [5]

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