Numerical solutions of equations – 2023 Nov CIE Mathematics

1. Nov/2023/Paper_9709/21/No.7

The curve with equation $e^{2x} - 18x + y^3 + y = 11$ has a stationary point at (p, q).

(a) Find the exact value of p. [4]

(b) Show that $q = \sqrt[3]{2 + 18 \ln 3 - q}$. [2]

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(c) Show by calculation that the value of q lies between 2.5 and 3.0. [2]

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(d) Use an iterative formula, based on the equation in **(b)**, to find the value of q correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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2. Nov/2023/Paper_9709/31/No.8

(a) By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root. [2]

(b) Show by calculation that this root lies between 1 and 2. [2]

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(c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln\left(3 + \sqrt{x_n}\right)$$

converges, then it converges to the root of the equation in **(a)**. [1]

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(d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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3. Nov/2023/Paper_9709/32/No.6

(a) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leqslant \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.6 and 0.8. [2]

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(c) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{1}{2-\cos x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]