Trigonometry – 2023 Nov CIE Mathematics

1. Nov/2023/Paper_9709/21/No.1

It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60°)$. [3]

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2. Nov/2023/Paper_9709/21/No.6

(a) Show that $\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) \equiv 4 + 6\cos\theta - 4\cos^2\theta$. [3]

(b) Solve the equation

$$\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) + 3 = 0$$

for $-\pi < \theta < 0$. [3]

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(c) Find $\int \operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta)\,d\theta$. [3]

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3. Nov/2023/Paper_9709/22/No.2

Solve the equation $\sec\theta\cos(\theta - 60°) = 4$ for $-180° < \theta < 180°$. [5]

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4. Nov/2023/Paper_9709/22/No.7

(a) Prove that $\sin 2x(\cot x + 3\tan x) \equiv 4 - 2\cos 2x$. [4]

(b) Hence find the exact value of $\cot \frac{1}{12}\pi + 3\tan \frac{1}{12}\pi$. [2]

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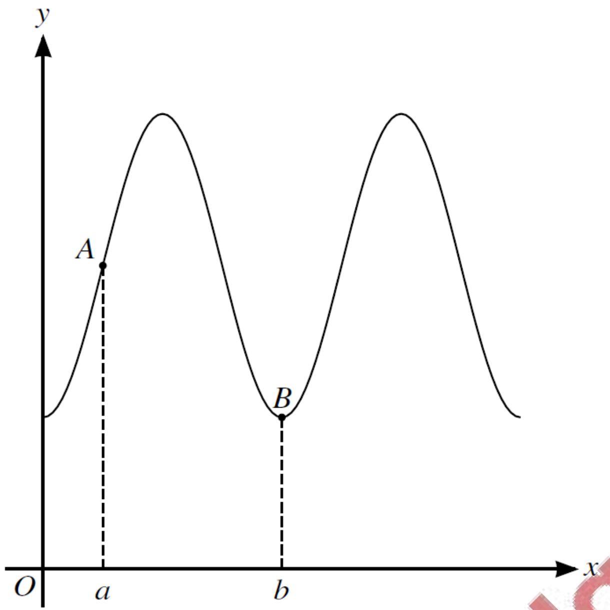
(c)



The diagram shows the curve with equation $y = 4 - 2\cos 2x$, for $0 < x < 2\pi$. At the point A, the gradient of the curve is 4. The point B is a minimum point. The x-coordinates of A and B are a and b respectively.

Show that $\int_a^b (4 - 2\cos 2x)\,dx = 3\pi + 1$. [5]

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5. Nov/2023/Paper_9709/31/No.5

(a) Given that

$$\sin(x + \tfrac{1}{6}\pi) - \sin(x - \tfrac{1}{6}\pi) = \cos(x + \tfrac{1}{3}\pi) - \cos(x - \tfrac{1}{3}\pi),$$

find the exact value of $\tan x$. [4]

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(b) Hence find the exact roots of the equation

$$\sin(x + \tfrac{1}{6}\pi) - \sin(x - \tfrac{1}{6}\pi) = \cos(x + \tfrac{1}{3}\pi) - \cos(x - \tfrac{1}{3}\pi)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

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6. Nov/2023/Paper_9709/32/No.7

(a) By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

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(b) Hence solve the equation

$$\cos 3\theta + \cos\theta \cos 2\theta = \cos^2\theta$$

for $0° \leqslant \theta \leqslant 180°$. [5]

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7. Nov/2023/Paper_9709/33/No.6

(a) Show that the equation $\cot^2\theta + 2\cos 2\theta = 4$ can be written in the form

$$4\sin^4\theta + 3\sin^2\theta - 1 = 0.$$ [3]

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(b) Hence solve the equation $\cot^2\theta + 2\cos 2\theta = 4$, for $0° < \theta < 360°$. [3]

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