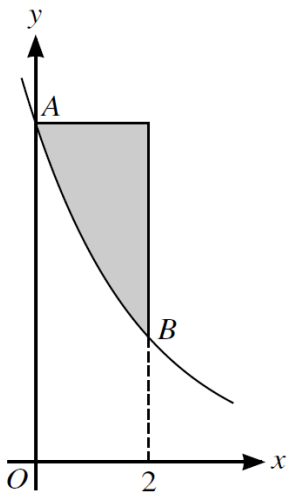
Integration – 2023 Nov CIE Mathematics

1. Nov/2023/Paper_9709/21/No.3

(a) Find $\int_4^{10} \frac{4}{2x-5}\,\mathrm{d}x$, giving your answer in the form $\ln a$, where a is an integer. [4]

(b) Find the exact value of $\int_4^{10} \mathrm{e}^{2x-5}\,\mathrm{d}x$. [2]

2. Nov/2023/Paper_9709/22/No.3



The diagram shows the curve with equation $y = 6e^{-\frac{1}{2}x}$. The points on the curve with x-coordinates 0 and 2 are denoted by A and B respectively. The shaded region is enclosed by the curve, the line through A parallel to the x-axis and the line through B parallel to the y-axis.

(a) Find the exact gradient of the curve at B. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact area of the shaded region. [3]

3. Nov/2023/Paper_9709/22/No.5

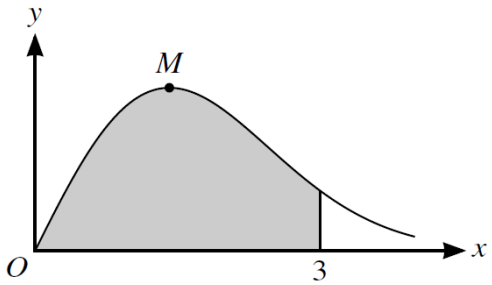
(a) Find the quotient when $6x^3 - 5x^2 - 24x - 4$ is divided by $(2x + 1)$, and show that the remainder is 6. [3]

(b) Hence find

$$\int_2^7 \frac{6x^3 - 5x^2 - 24x - 4}{2x + 1}\,\mathrm{d}x,$$

giving your answer in the form $a + \ln b$, where a and b are integers. [5]

4. Nov/2023/Paper_9709/31/No.9



The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \geqslant 0$, and its maximum point M.

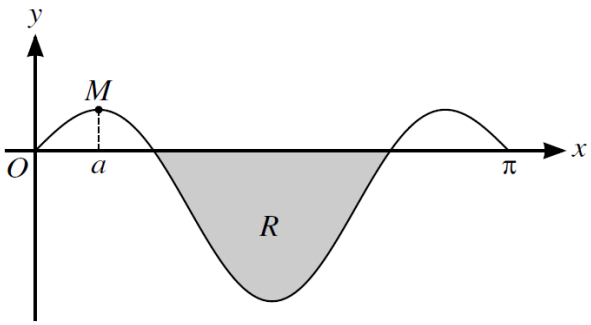
(a) Find the exact coordinates of M. [4]

(b) Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 3$. [5]

5. Nov/2023/Paper_9709/32/No.5

Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4}\,\mathrm{d}x$. [6]

6. Nov/2023/Paper_9709/32/No.9

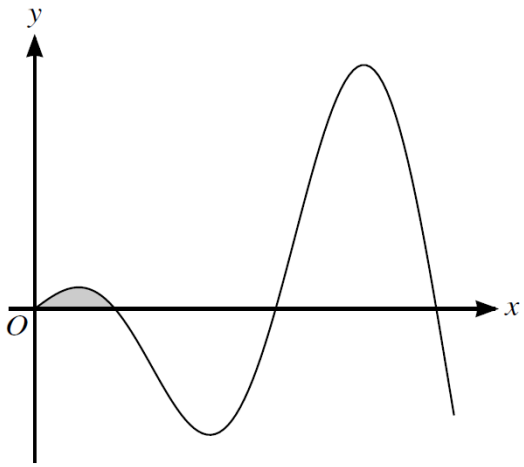


The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leqslant x \leqslant \pi$, and a maximum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Find the value of a correct to 2 decimal places. [5]

(b) Find the exact area of the region R, giving your answer in simplified form. [4]

7. Nov/2023/Paper_9709/33/No.10



The diagram shows the curve $y = x\cos 2x$, for $x \geqslant 0$.

(a) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}\pi$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x-axis. [5]