Hypothesis tests – 2023 Nov CIE Mathematics

1. Nov/2023/Paper_9709/61/No.5

In the past the number of enquiries per minute at a customer service desk has been modelled by a random variable with distribution Po(0.31). Following a change in the position of the desk, it is expected that the mean number of enquiries per minute will increase. In order to test whether this is the case, the total number of enquiries during a randomly chosen 5-minute period is noted. You should assume that a Poisson model is still appropriate.

Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

2. Nov/2023/Paper_9709/61/No.7

A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

3. Nov/2023/Paper_9709/62/No.3

A researcher read a magazine article which stated that boys aged 1 to 3 prefer green to orange. It claimed that, when offered a green cube and an orange cube to play with, a boy is more likely to choose the green one.

The researcher disagrees with this claim. She believes that boys of this age are equally likely to choose either colour. In order to test her belief, the researcher carried out a hypothesis test at the 5% significance level. She offered a green cube and an orange cube to each of 10 randomly chosen boys aged 1 to 3, and recorded the number, X, of boys who chose the green cube.

Out of the 10 boys, 8 boys chose the green cube.

(a) **(i)** Assuming that the researcher's belief that either colour cube is equally likely to be chosen is valid, a student correctly calculates that $P(X = 8) = 0.0439$, correct to 3 significant figures. He says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. [1]

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(ii) Carry out the test on the researcher's claim that either colour cube is equally likely to be chosen. [5]

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(b) Another researcher claims that a Type I error was made in carrying out the test.

Explain why this cannot be true. [1]

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A similar test, at the 5% significance level, was carried out later using 10 other randomly chosen boys aged 1 to 3.

(c) Find the probability of a Type I error. [2]

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4. Nov/2023/Paper_9709/62/No.4

The height H, in metres, of mature trees of a certain variety is normally distributed with standard deviation 0.67. In order to test whether the population mean of H is greater than 4.23, the heights of a random sample of 200 trees are measured.

(a) Write down suitable null and alternative hypotheses for the test. [1]

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The sample mean height, $\bar{h}$ metres, of the 200 trees is found and the test is carried out. The result of the test is to reject the null hypothesis at the 5% significance level.

(b) Find the set of possible values of $\bar{h}$. [3]

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(c) Ajit said, 'In **(b)** we had to assume that $\overline{H}$ is normally distributed, so it was necessary to use the Central Limit Theorem.'

Explain whether you agree with Ajit. [1]

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