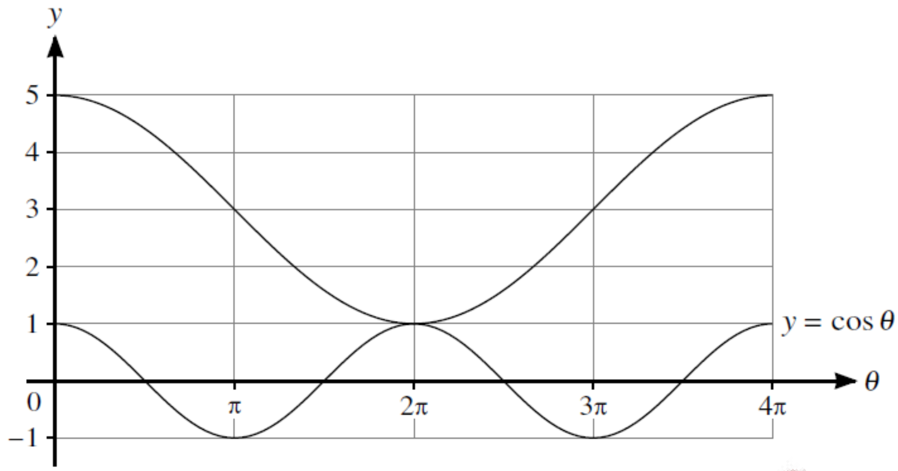
Functions and transformations – 2020 AS

1. Nov/2020/Paper_9709/11/No.4



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

2. Nov/2020/Paper_9709/11/No.11

The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $fg(x)$. [1]

(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

3. Nov/2020/Paper_9709/12/No.5

Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

4. Nov/2020/Paper_9709/12/No.11

A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

....

....

(ii) $k = 1$ [1]

....

....

(iii) $k = 3$ [1]

....

....

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

5. Nov/2020/Paper_9709/13/No.1b

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

6. Nov/2020/Paper_9709/13/No.6

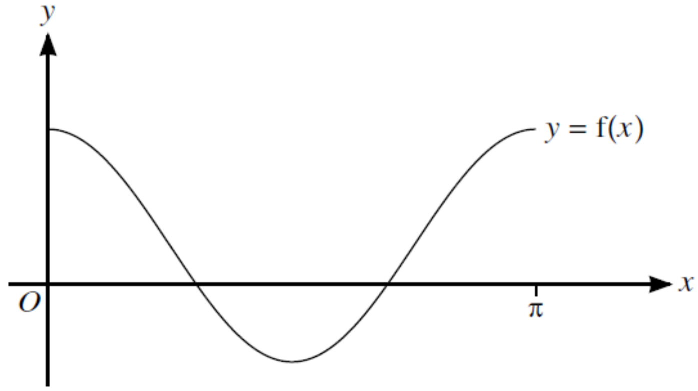
The function f is defined by $f(x) = \frac{2x}{3x - 1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $\frac{2}{3} + \frac{2}{3(3x - 1)}$ can be expressed as $\frac{2x}{3x - 1}$. [2]

(c) State the range of f. [1]

7. June/2020/Paper_9709/11/No.4



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = g(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

8. June/2020/Paper_9709/11/No.6

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - a,$$
$$g : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b. [4]

(b) Using these values of a and b, find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

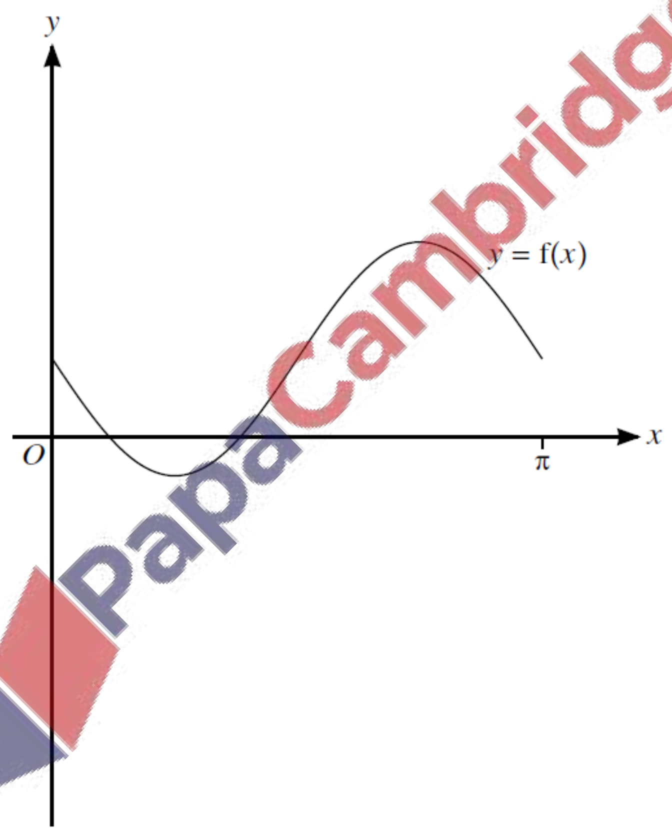
9. June/2020/Paper_9709/12/No.9

Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

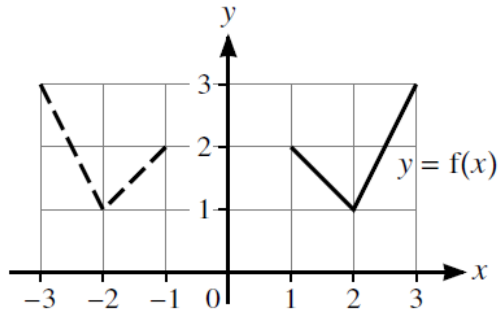
$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

10. June/2020/Paper_9709/13/No.3

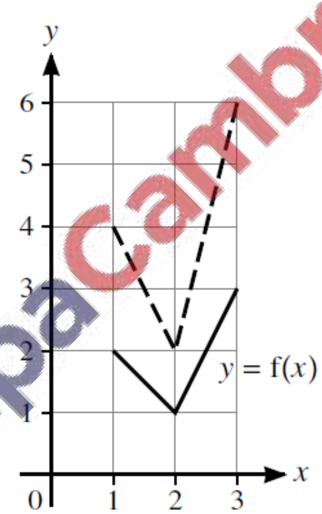
In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)



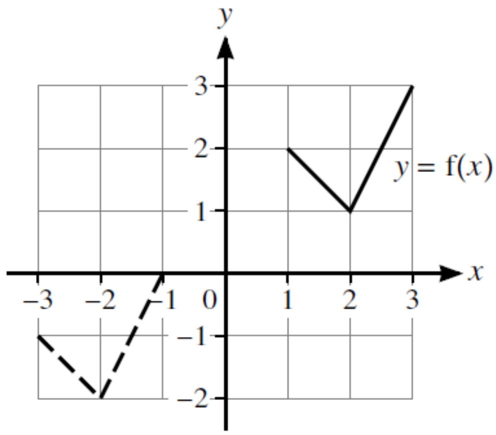
State, in terms of f, the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

11. June/2020/Paper_9709/13/No.9

The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express f(x) in the form $(x - a)^2 + b$. [2]

It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(d) Find an expression for gf(x) and state the range of gf. [3]

12. March/2020/Paper_9709/12/No.1

The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

13. March/2020/Paper_9709/12/No.2

The graph of $y = f(x)$ is transformed to the graph of $y = 1 + f(\frac{1}{2}x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

14. March/2020/Paper_9709/12/No.9b-9d

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]