Series and Binomial Expansion – 2020 AS

1. Nov/2020/Paper_9709/11/No.5

In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

2. Nov/2020/Paper_9709/11/No.8

A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

3. Nov/2020/Paper_9709/12/No.1

The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

4. Nov/2020/Paper_9709/12/No.2

The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

5. Nov/2020/Paper_9709/12/No.4

The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

6. Nov/2020/Paper_9709/13/No.5

In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

7. Nov/2020/Paper_9709/13/No.7

The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

8. June/2020/Paper_9709/11/No.1

The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

9. June/2020/Paper_9709/11/No.2

The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx+\frac{1}{x}\right)^5+\left(1-\frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

10. June/2020/Paper_9709/11/No.3

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

11. June/2020/Paper_9709/12/No.1

(a) Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

12. June/2020/Paper_9709/12/No.4

The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

13. June/2020/Paper_9709/13/No.4

(a) Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

14. March/2020/Paper_9709/12/No.6

The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

15. March/2020/Paper_9709/12/No.8

A woman's basic salary for her first year with a particular company is $30 000 and at the end of the year she also gets a bonus of $600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by $100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]