Trigonometry – 2020 AS
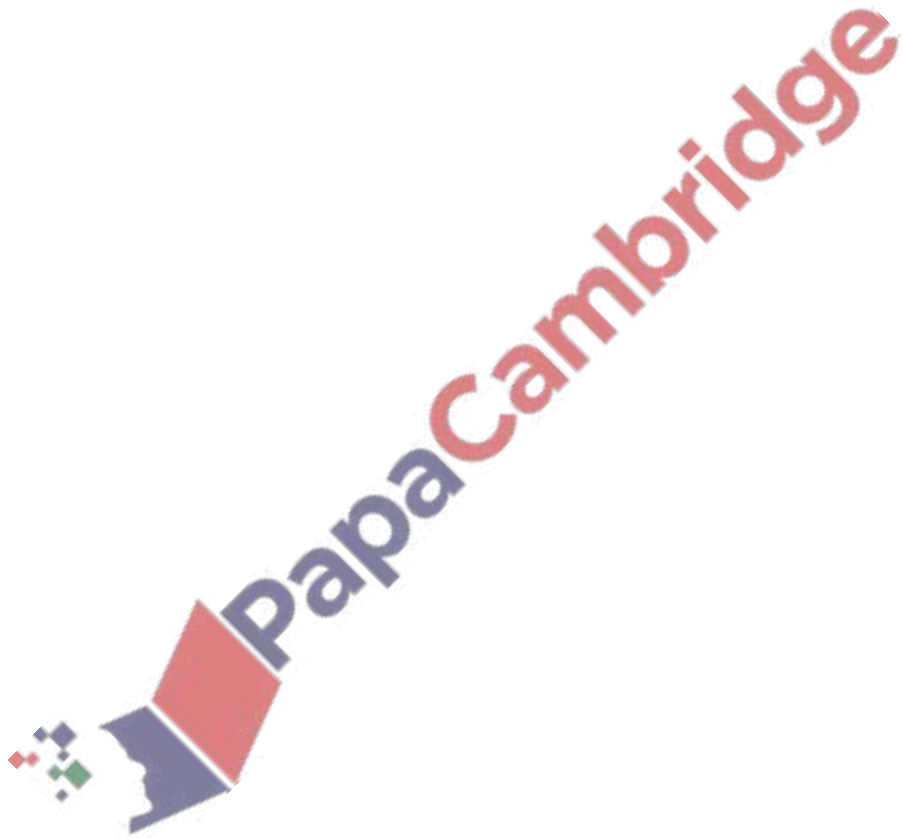
1.

(a) Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta.$ [3]

(b) Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

2.
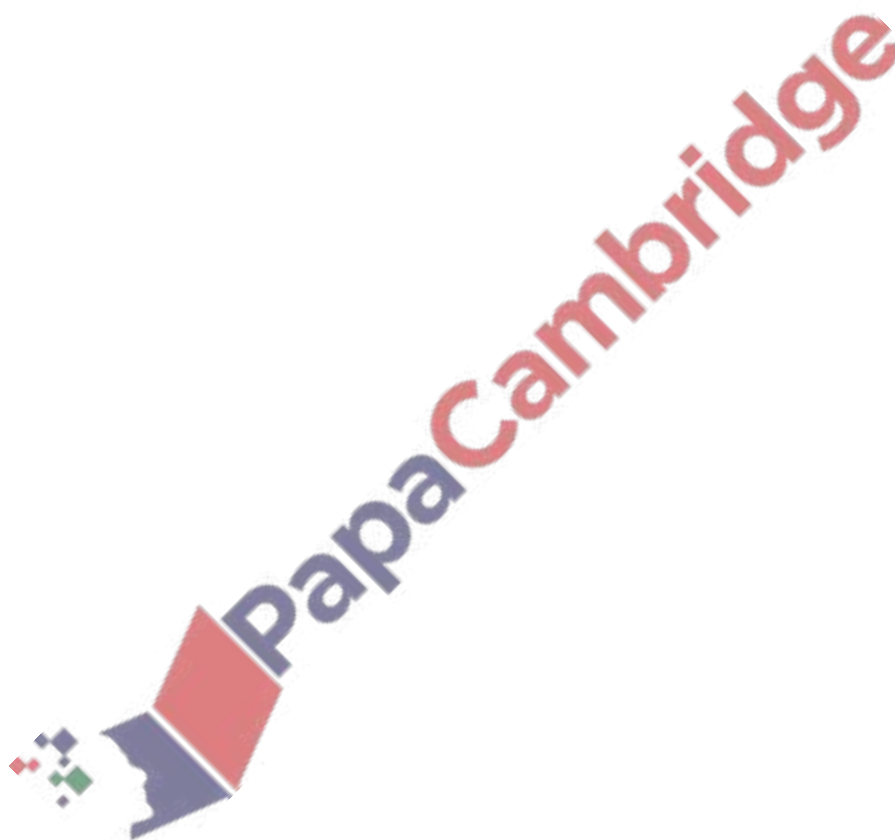
(a) Prove the identity $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) \equiv \dfrac{1}{\tan x}$. [4]
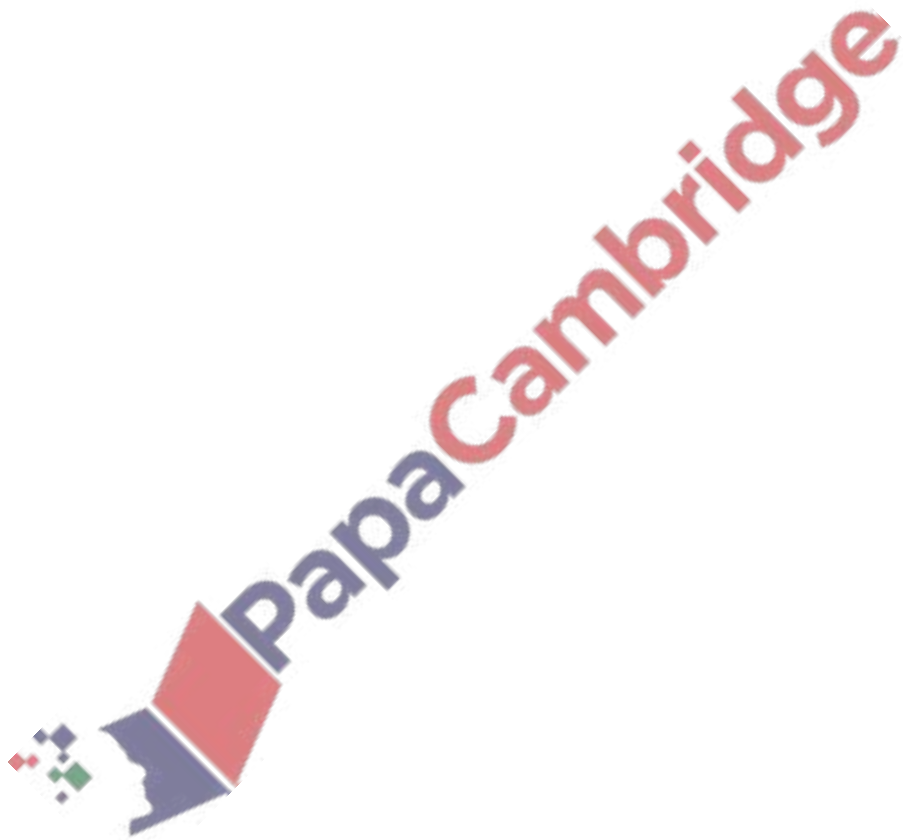
(b) Hence solve the equation $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]
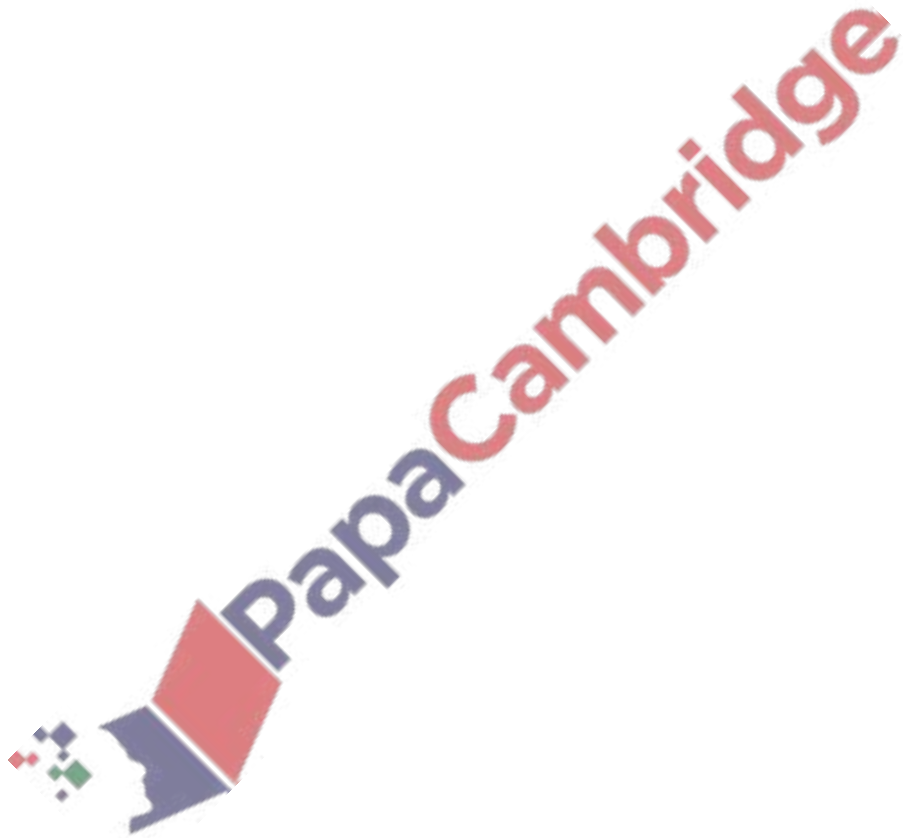
3.

Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

4.

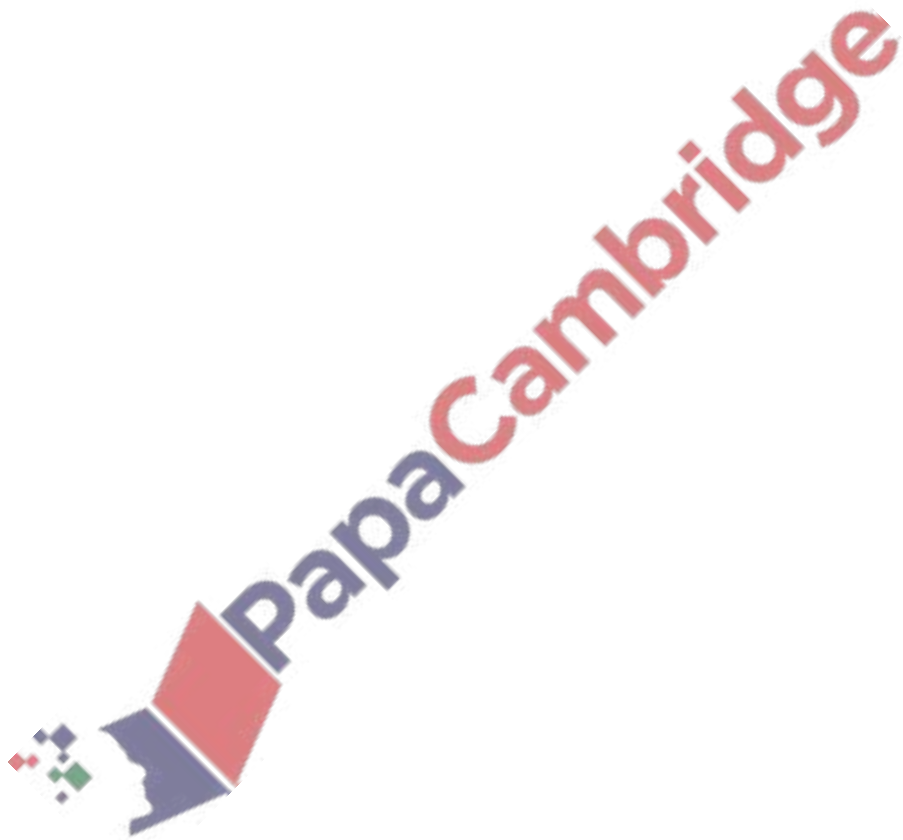(a) Prove the identity $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}\equiv\dfrac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{3}{\sin\theta}$, for $0\leqslant\theta\leqslant 2\pi$. [3]
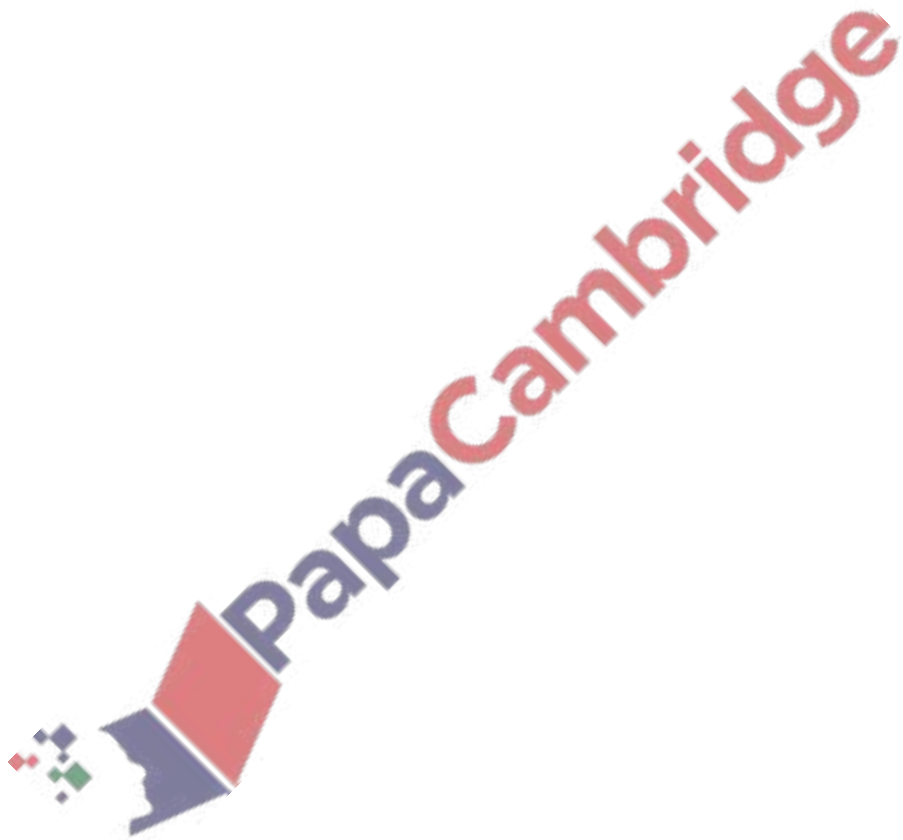
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5.

    (a) Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$.   [3]

    (b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$.   [2]

(a) Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

7.

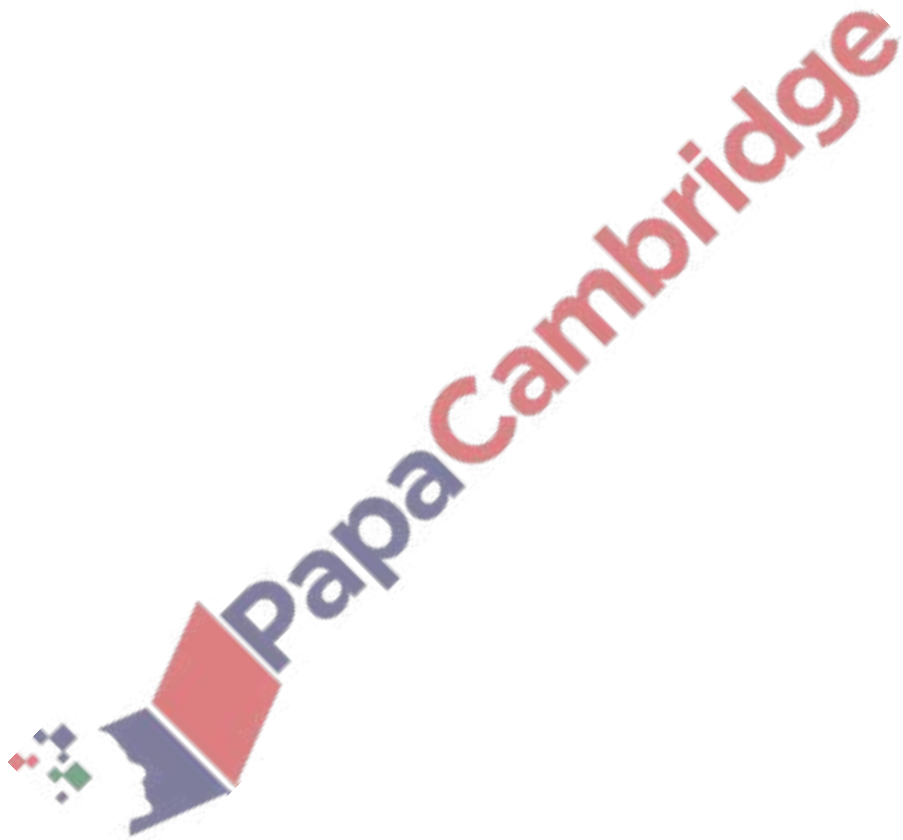Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

8.

(a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]