Differentiation and Integration – 2021 AS

1. June/2021/Paper_9709/11/No.1

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

Find the equation of the curve. [4]

2. June/2021/Paper_9709/11/No.6

The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

3. June/2021/Paper_9709/11/No.11

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

4. June/2021/Paper_9709/12/No.3

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2,\ k)$ $B\,(2.9,\ 2.8025)$ $C\,(2.99,\ 2.9800)$ $D\,(2.999,\ 2.9980)$ $E\,(3,\ 3)$

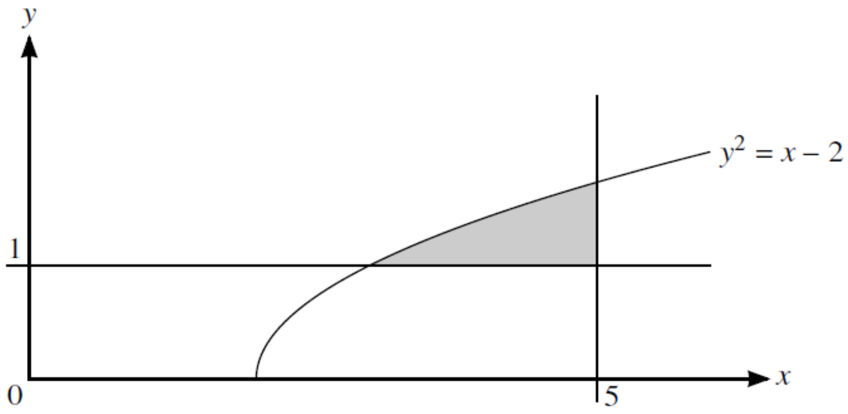
(a) Find k, giving your answer correct to 4 decimal places. [1]

(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

5. June/2021/Paper_9709/12/No.9



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

6. June/2021/Paper_9709/12/No.11

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

(b) Find the equation of the curve. [4]

(c) Find $\frac{d^2y}{dx^2}$. [2]

(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

7. June/2021/Paper_9709/13/No.1

A curve with equation $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 6x^2 - \dfrac{8}{x^2}$. It is given that the curve passes through the point (2, 7).

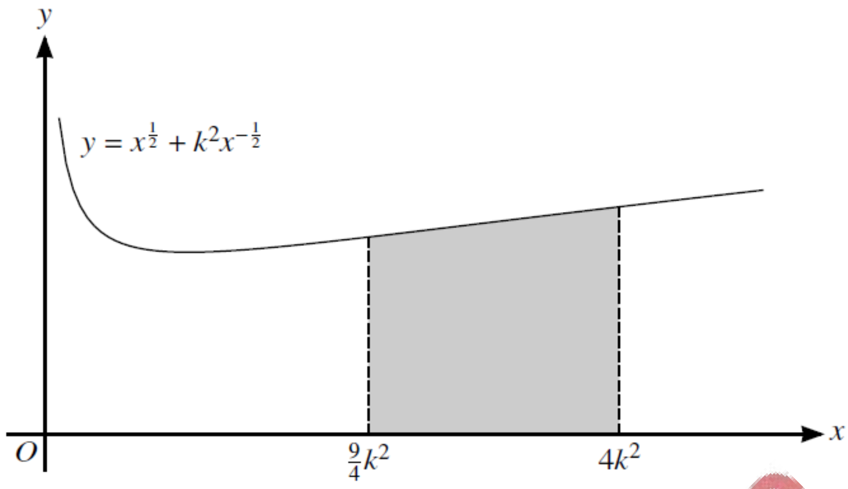
Find $\mathrm{f}(x)$. [3]

8. June/2021/Paper_9709/13/No.2

The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

9. June/2021/Paper_9709/13/No.13



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

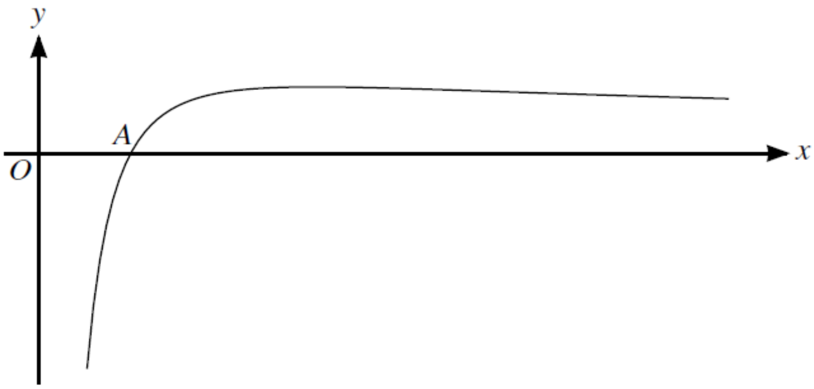
10. March/2021/Paper_9709/12/No.6

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1,\ -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

11. March/2021/Paper_9709/12/No.11



The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

(b) Find the equation of the tangent to the curve at A. [4]

(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]