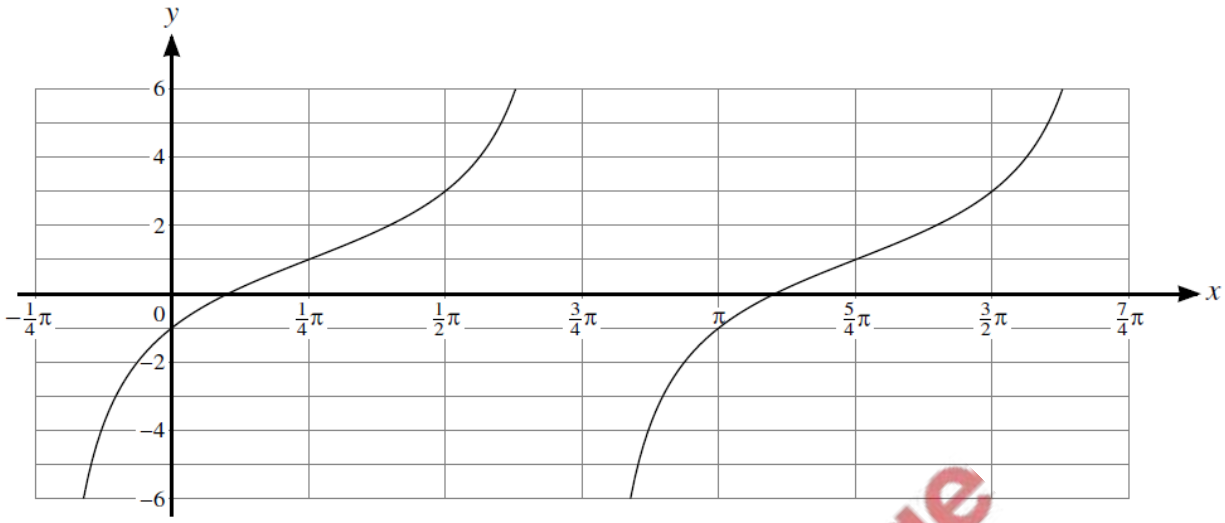
Functions and Transformation– 2021 AS

1. June/2021/Paper_9709/11/No.4



The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

2. June/2021/Paper_9709/11/No.9

Functions f and g are defined as follows:

$$f(x) = (x - 2)^2 - 4 \text{ for } x \geqslant 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

(b) Find $f^{-1}(x)$. [2]

(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

(d) Given instead that $ggf^{-1}(12) = 62$, find the possible values of a. [5]

3. June/2021/Paper_9709/12/No.2

(a) The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

4. June/2021/Paper_9709/12/No.5

The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

5. June/2021/Paper_9709/13/No.6

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

6. June/2021/Paper_9709/13/No.8

Functions f and g are defined as follows:

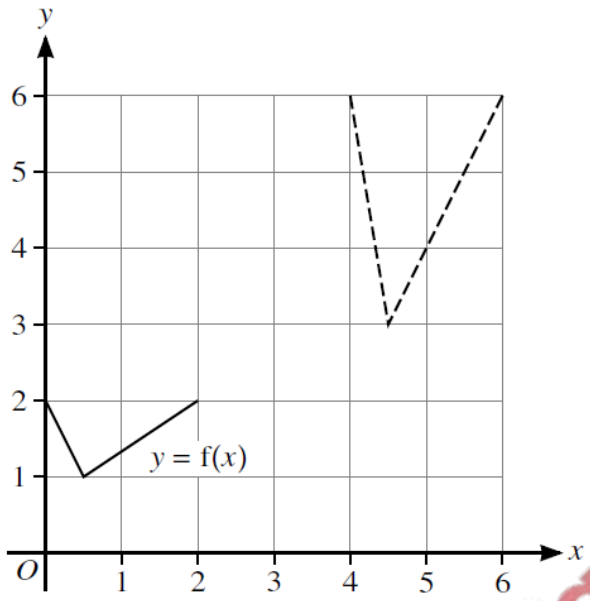
$$\mathrm{f} : x \mapsto x^2 - 1 \text{ for } x < 0,$$

$$\mathrm{g} : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $\mathrm{fg}(x) = 3$. [4]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

7. March/2021/Paper_9709/12/No.5



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

(a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

8. March/2021/Paper_9709/12/No.7

Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\mathrm{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(c) Solve the equation $\mathrm{gf}(x) = 13$. [3]