1. Nov/2021/Paper_9709/11/No.9

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

(b) Find the coordinates of the stationary points on the curve. [5]

(c) Find $f''(x)$. [1]

(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

2. Nov/2021/Paper_9709/11/No.10

(a) Find $\displaystyle\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]

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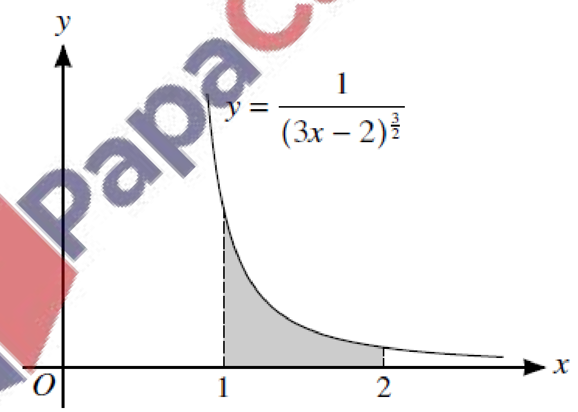

The diagram shows the curve with equation $y = \dfrac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

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The normal to the curve at the point (1, 1) crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

3. Nov/2021/Paper_9709/12/4

A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $(2, 5\frac{2}{3})$.

Find the equation of the curve. [4]

4. Nov/2021/Paper_9709/12/9

The volume V m^3 of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of 1.5 m^3 per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

5. Nov/2021/Paper_9709/12/10

The function f is defined by $f(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

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(c) Given that this is the only stationary point of the curve, find the range of f. [2]

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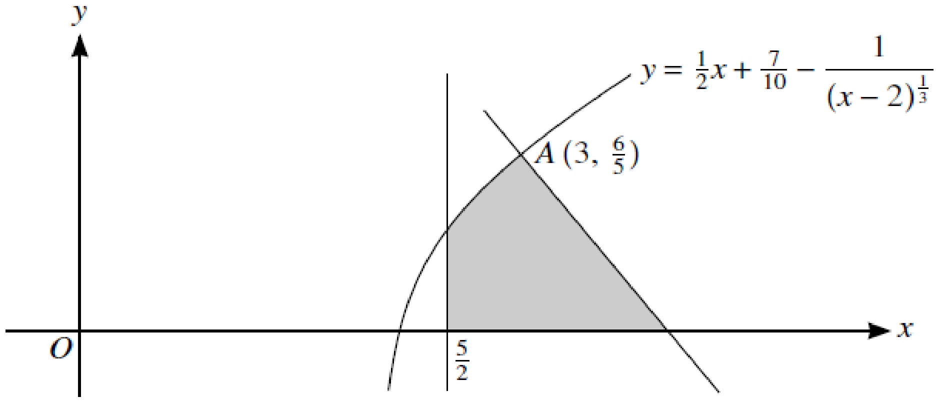
6. Nov/2021/Paper_9709/12/11



The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\ (3, \frac{6}{5})$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

7. Nov/2021/Paper_9709/13/3(b)

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

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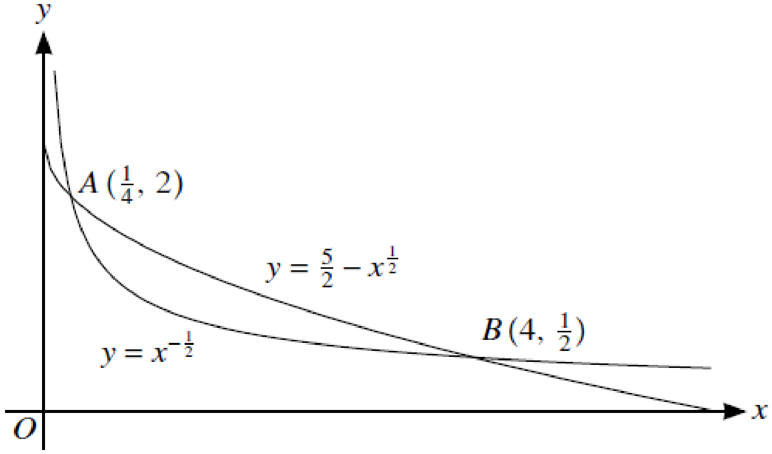
8. Nov/2021/Paper_9709/13/8



The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1,\ 1)$ intersects the y-axis at the point $(0,\ p)$.

Find the value of p. [4]

9. Nov/2021/Paper_9709/13/10

A curve has equation $y = f(x)$ and it is given that

$$f'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

(a) Find $f''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

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It is now given that $k = -3$ and the minimum point is at $(2, 3\tfrac{1}{2})$.

(b) Find $f(x)$. [4]

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(c) Find the coordinates of the other stationary point and determine its nature. [4]