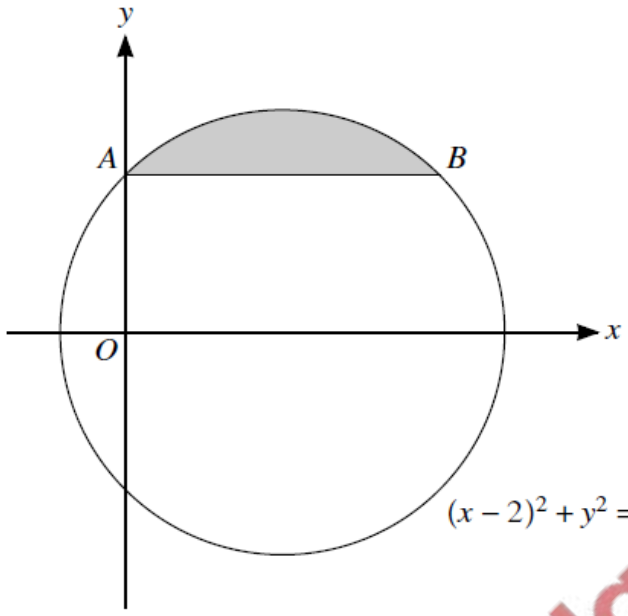
Differentiation and Integration – 2022 AS Nov

1. March/2022/Paper_9709/12/No.1

A curve with equation $y = f(x)$ is such that $f'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $f(8) = 5$.

Find $f(x)$. [4]

2. March/2022/Paper_9709/12/No.8



The diagram shows the circle with equation $(x-2)^2+y^2=8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

3. March/2022/Paper_9709/12/No.11

It is given that a curve has equation $y = k(3x - k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

The function f has a stationary value at $x = a$ and is defined by

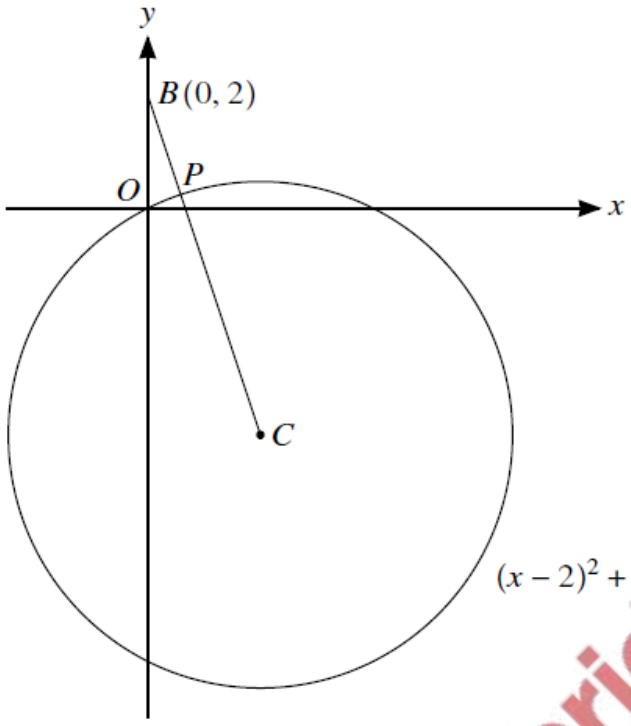
$$f(x) = 4(3x - 4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

(c) The function g is defined by $g(x) = -(3x + 1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

4. June/2022/Paper_9709/11/No.7



The diagram shows the circle with equation $(x-2)^2+(y+4)^2=20$ and with centre C. The point B has coordinates $(0, 2)$ and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

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(b) Hence find the coordinates of P, giving your answer in exact form. [5]

5. June/2022/Paper_9709/11/No.10

The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^\infty f(x)\,dx$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

6. June/2022/Paper_9709/12/No.3

The equation of a curve is such that $\frac{dy}{dx} = 3(4x - 7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, \frac{5}{2})$.

Find the equation of the curve. [4]

7. June/2022/Paper_9709/12/No.6

The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

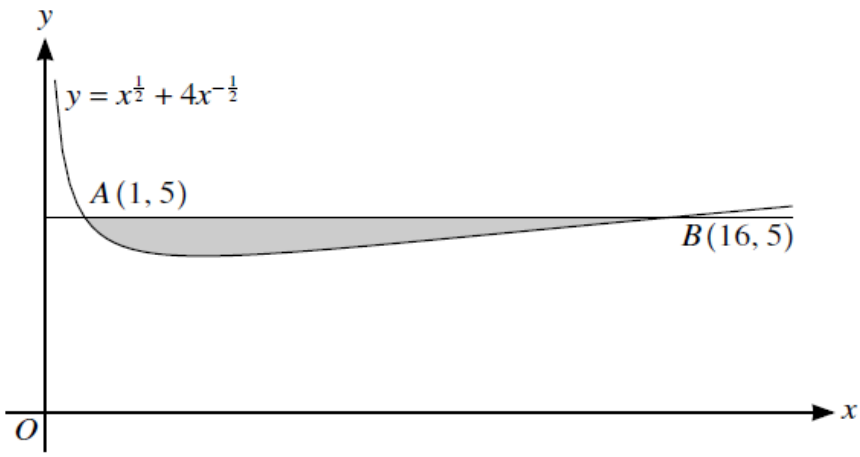
8. June/2022/Paper_9709/12/No.9

The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point of the curve and determine its nature. [4]

9. June/2022/Paper_9709/13/No.8



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1, 5)$ and $B\,(16, 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

(b) Calculate the area of the shaded region. [4]

10. June/2022/Paper_9709/13/No.10

The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^\infty f(x)\,dx$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]