Functions – 2022 AS June

1. March/2022/Paper_9709/12/No.5(b)

(b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

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2. March/2022/Paper_9709/12/No.9

Functions f, g and h are defined as follows:

$$\text{f} : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$

$$\text{g} : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$

$$\text{h} : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $\text{f}(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $f(x) \equiv gh(x)$, find the values of m and n. [4]

3. June/2022/Paper_9709/11/No.6

The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $1 - \frac{8}{x^2+4}$ can be expressed as $\frac{x^2-4}{x^2+4}$ and hence state the range of f. [4]

(c) Explain why the composite function ff cannot be formed. [1]

4. June/2022/Paper_9709/11/No.8

(a) The curve $y = \sin x$ is transformed to the curve $y = 4\sin(\frac{1}{2}x - 30°)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

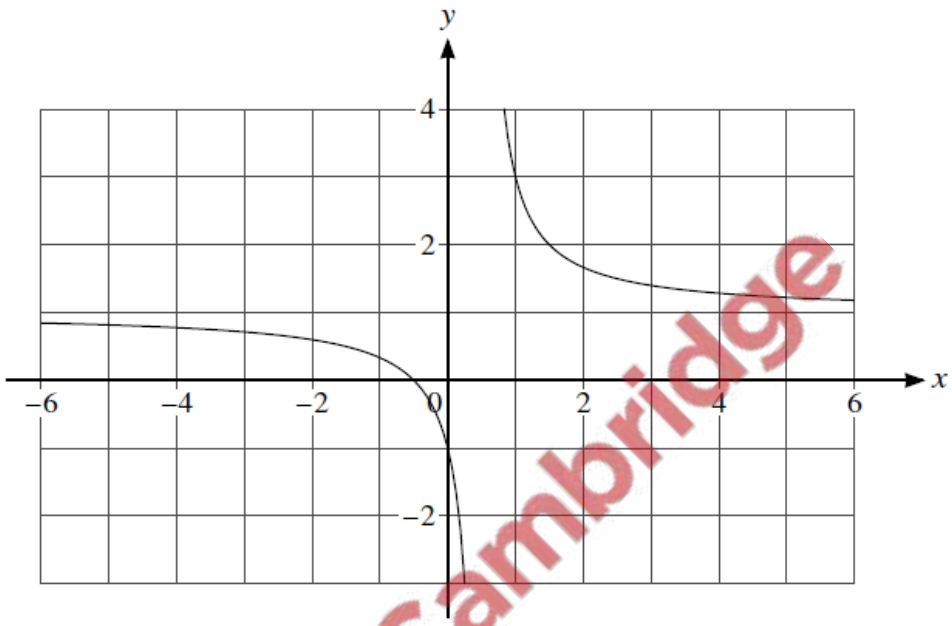
5. June/2022/Paper_9709/12/No.10

Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1}. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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(c) Find $gf^{-1}(3)$. [2]

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(d) Explain why $g^{-1}(x)$ cannot be found. [1]

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(e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

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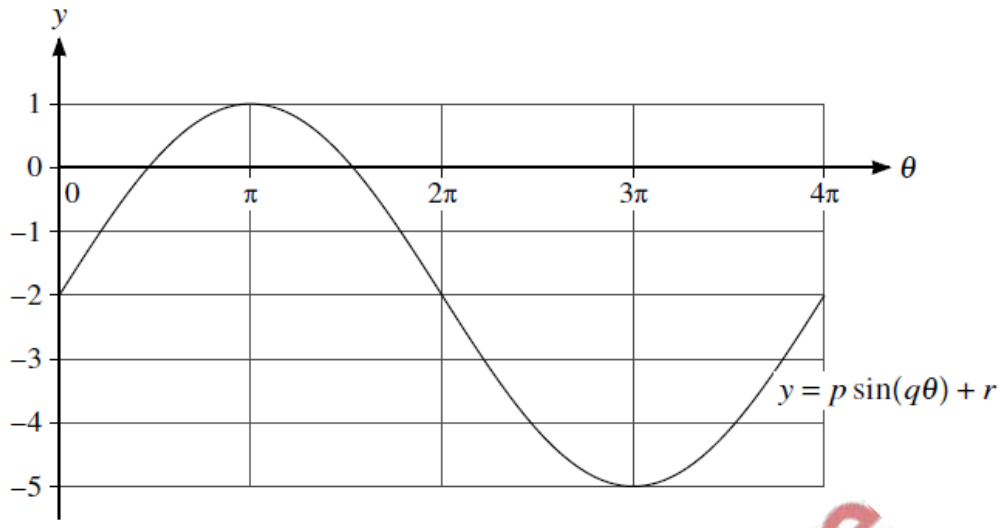
6. June/2022/Paper_9709/13/No.2



The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.

(a) State the value of p. [1]

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(b) State the value of q. [1]

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(c) State the value of r. [1]

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7. June/2022/Paper_9709/13/No.4

(a) The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

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(b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

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8. June/2022/Paper_9709/13/No.6

The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]