1. Nov/2022/Paper_9709_11/No.8

The function f is defined by $f(x) = 2 - \frac{3}{4x-p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx - d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

2. Nov/2022/Paper_9709_11/No.9

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]

(b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

(c) Express $g(x)$ in the form $kf(x+h)$, where k and h are integers. [1]

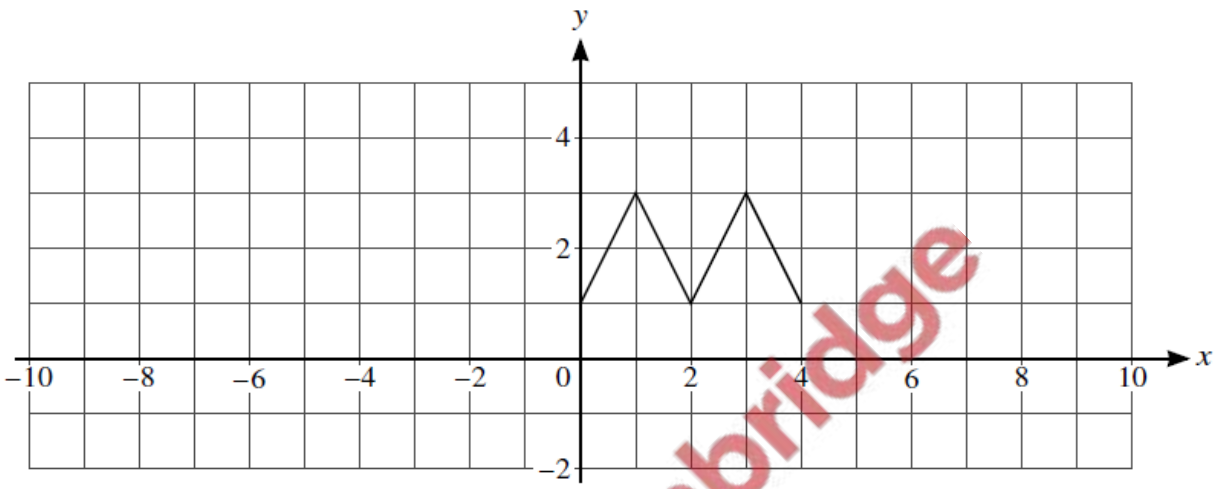
(d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

3. Nov/2022/Paper_9709_12/No.5

The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

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4. Nov/2022/Paper_9709_12/No.9

Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $gf(x)$. [1]

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(b) Given that $gf(2) = 11$, find the value of a. [2]

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(c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

(e) Explain why the composite function fg cannot be formed. [1]

5. Nov/2022/Paper_9709_13/No.2

The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

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(b) Find the range of f. [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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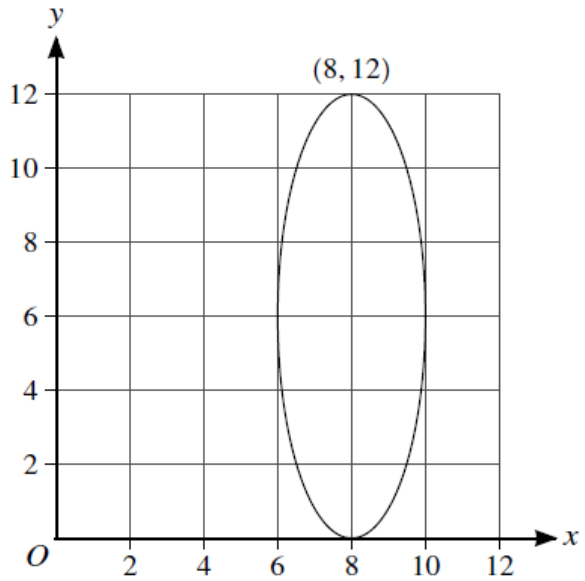
6. Nov/2022/Paper_9709_13/No.5



The diagram shows a curve which has a maximum point at (8, 12) and a minimum point at (8, 0). The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y-direction.

(a) State the scale factor of the stretch. [1]

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(b) State the radius of the original circle. [1]

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(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

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(d) State the coordinates of the centre of the original circle. [2]

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