1.

   The coefficient of x^2 in the expansion of $\left(1 + \dfrac{2}{p}x\right)^5 + (1 + px)^6$ is 70.

   Find the possible values of the constant p.   [6]

A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground.

[3]

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

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(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

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The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

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4.

A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term. [4]

A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is $4\,\text{m}$, giving your answer in cm per minute. [5]

6.

The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. [6]