1. June/2023/Paper_9709/11/No.9

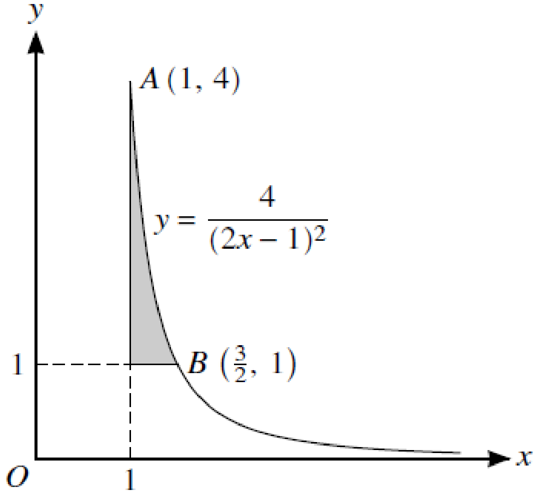
Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25+h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

2. June/2023/Paper_9709/11/No.10



The diagram shows part of the curve with equation $y = \dfrac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\ (1,\ 4)$ and $B,\ \left(\frac{3}{2},\ 1\right)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

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(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

3. June/2023/Paper_9709/11/No.11

The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

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(b) Determine the nature of this stationary point. [2]

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(c) Find the equation of the curve. [3]

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(d) Find the coordinates of any other stationary points on the curve. [2]

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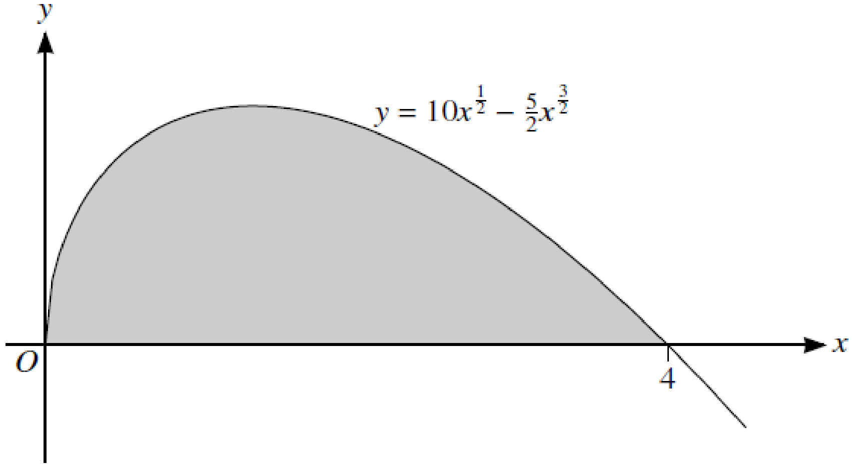
4. June/2023/Paper_9709/12/No.1

The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

5. June/2023/Paper_9709/12/No.5



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$.

Find the area of the shaded region. [4]

6. June/2023/Paper_9709/12/No.11

The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$. [2]

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(b) Find the x-coordinate of the stationary point in terms of k. [2]

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(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

7. June/2023/Paper_9709/13/No.9

A curve which passes through (0, 3) has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 1 - \dfrac{2}{(x-1)^3}$.

(a) Find the equation of the curve. [4]

The tangent to the curve at (0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x+1)(x-1)^2 - 1 = 0$. [4]

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(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

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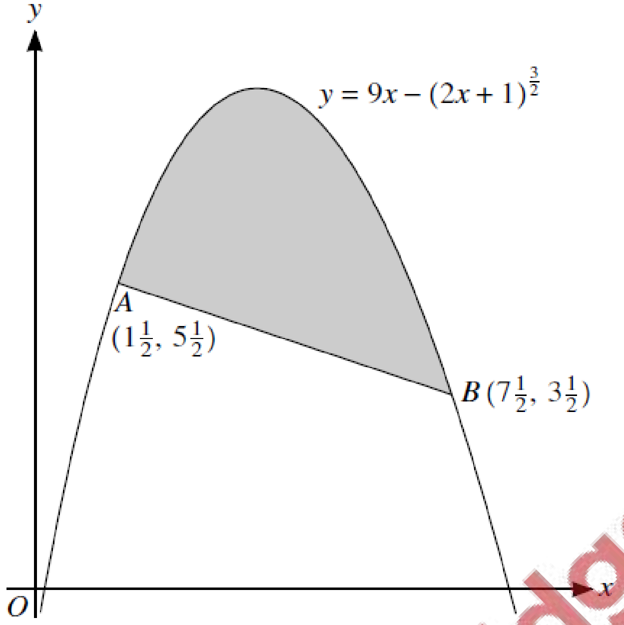
8. June/2023/Paper_9709/13/No.10



The diagram shows the points $A\,(1\frac{1}{2}, 5\frac{1}{2})$ and $B\,(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

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(b) Verify that the line AB is the normal to the curve at A. [3]

(c) Find the area of the shaded region. [5]