

2. June/2023/Paper_9709/11/No.7

A curve has equation $y = 2 + 3 \sin \frac{1}{2}x$ for $0 \leq x \leq 4\pi$.

(a) State greatest and least values of y .

[2]

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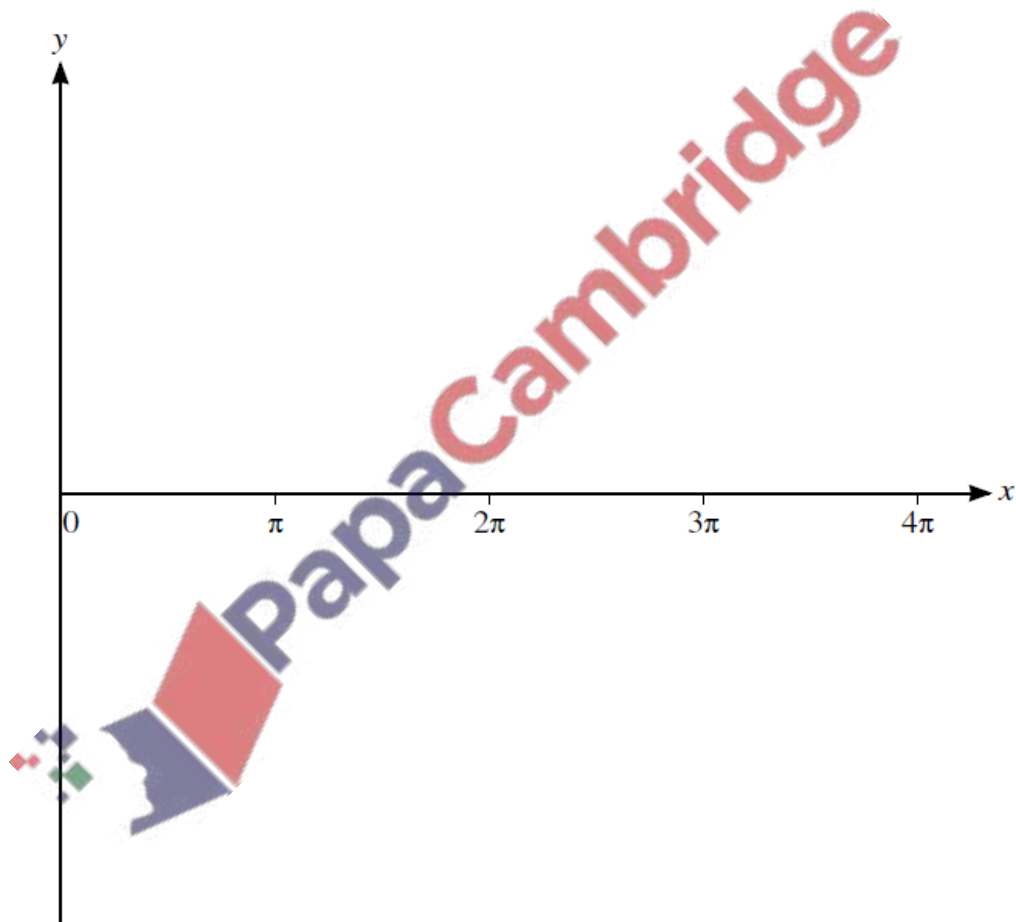
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(b) Sketch the curve.

[2]



(c) State the number of solutions of the equation

$$2 + 3 \sin \frac{1}{2}x = 5 - 2x$$

for $0 \leq x \leq 4\pi$.

[1]

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(a) (i) By first expanding $(\cos \theta + \sin \theta)^2$, find the three solutions of the equation

$$(\cos \theta + \sin \theta)^2 = 1$$

for $0 \leq \theta \leq \pi$.

[3]

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(ii) Hence verify that the only solutions of the equation $\cos \theta + \sin \theta = 1$ for $0 \leq \theta \leq \pi$ are 0 and $\frac{1}{2}\pi$. [2]



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- (b) Prove the identity $\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} \equiv \frac{\cos \theta + \sin \theta - 1}{1 - 2 \sin^2 \theta}$. [3]

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- (c) Using the results of (a)(ii) and (b), solve the equation

$$\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} = 2(\cos \theta + \sin \theta - 1)$$

for $0 \leq \theta \leq \pi$. [3]

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(a) Show that the equation

$$3 \tan^2 x - 3 \sin^2 x - 4 = 0$$

may be expressed in the form $a \cos^4 x + b \cos^2 x + c = 0$, where a , b and c are constants to be found. [3]

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(b) Hence solve the equation $3 \tan^2 x - 3 \sin^2 x - 4 = 0$ for $0^\circ \leq x \leq 180^\circ$. [4]

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