Trigonometry – 2023 AS Mathematics 9709

1. Nov/2023/Paper_9709/11/No.5

(a) Show that the equation

$$4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$$

may be expressed in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers to be found. [3]

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(b) Hence solve the equation $4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$ for $0° \leqslant x \leqslant 360°$. [3]

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2. Nov/2023/Paper_9709/12/No.2

Find the exact solution of the equation

$$\tfrac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}\left(\tfrac{1}{2}\sqrt{3}\right). \qquad [2]$$

3. Nov/2023/Paper_9709/12/No.7

(a) Verify the identity $(2x-1)(4x^2+2x-1) \equiv 8x^3-4x+1$. [1]

(b) Prove the identity $\dfrac{\tan^2\theta+1}{\tan^2\theta-1} \equiv \dfrac{1}{1-2\cos^2\theta}$. [3]

(c) Using the results of **(a)** and **(b)**, solve the equation

$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 4\cos\theta,$$

for $0° \leqslant \theta \leqslant 180°$. [5]

4. Nov/2023/Paper_9709/13/No.3

(a) Show that the equation

$$5\cos\theta - \sin\theta\tan\theta + 1 = 0$$

may be expressed in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $5\cos\theta - \sin\theta\tan\theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

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5. March/2023/Paper_9709/12/No.7

(a) By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta \sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]