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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01
Paper 1

General comments

The overall quality of work submitted in response to this Paper was good and provided clear evidence of a well-prepared candidature. Most candidates made a serious attempt at all the questions, submitted responses in order and set out their work in a clear way. The only opportunity for rubric infringement occurred in **Question 11**, but very few candidates wasted time by submitting responses to both options of that question.

There were some misreads, especially in **Question 10**, and quite a lot of elementary arithmetic and algebraic errors. Such errors can lead to severe consequences for a question response, for not only do they necessarily lead to incorrect results, but also, they can in certain situations, drive the candidate into unworkable, or at least time consuming, strategies. In order, therefore, to avoid these unfortunate situations, which in extreme cases can seriously undermine the overall examination performance, it is essential that work is checked at each stage of its development.

A feature of this new syllabus is that candidates must answer all questions if they are to obtain full credit. In this case, it is good to be able to record that with the exception of the vector product and linear spaces almost all candidates gave evidence of having an in depth knowledge of the entire syllabus. In particular, knowledge of complex numbers, tested in **Question 7**, was good. As in the case of previous A Level Further Mathematics Paper 1 examinations, the calculus topics, *per se,* were well understood and failures in questions, such as **4**, **5**, **6**, and **8** which related to this material were usually due to non-calculus errors.

Comments on specific questions

Question 1

This short introductory question did not go as well as expected. It contained a somewhat unusual element and this seems to have baffled most of the candidature.

Almost all candidates obtained a correct result for $S_n = \sum_{n=1}^{N} u_n$, by application of the difference method. A few treated this series as the difference of two geometric series and so obtained a correct but unsimplified result for S_n and this, more often than not, led to difficulties in the final part of the question.

The majority stated that the given series is convergent if $x < 0$, but made no mention of $x = 0$. Likewise, most stated, or implied, that $S_\infty = e^x$ for $x < 0$, but very few identified $S_\infty = 0$ when $x = 0$.

Answers: $S_N = e^x - e^{(N+1)x}$; infinite series is convergent for $x < 0$; $S_\infty = e^x$, for $x < 0$, $= 0$ for $x = 0$.

Question 2

In general this question was well answered.

Almost all candidates began by substituting $\frac{1}{y}$ for x in the given polynomial equation, and so went on to obtain the correct polynomial equation in y. As often as not, however, the terms of this equation were not ordered in the standard way. For the rest, the majority validly obtained $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1 - 2A$ and $\alpha^{-2} + \beta^{-2} + \gamma^{-2} + \delta^{-2} = 4 + A$, and so generally went on to obtain the correct value of A. Nevertheless there were some who, apparently, were unable to solve the equation $1 - 2A = 4 + A$ correctly. However, there were some who attempted to evaluate the second of these sums in terms of A by considering the coefficients of the x-equation. Although, in principle, this complicated strategy is feasible, hardly any candidates had the necessary algebraic expertise to argue successfully in this way.

Answer: $A = -1$.

Question 3

There were many essentially correct responses to this question, but only a minority of these were complete.

Most responses showed the simplification of $\phi = a_{n+1} - a_n$ to a form which clearly indicates divisibility by 24.

For the rest of the question, the main deficiencies were the general failure to state at the outset what the inductive hypothesis actually is, failure to verify that $24 \mid \phi_0$ (very often it was shown instead that $24 \mid a_1$), failure to establish the key result, namely, that $24 \mid \phi_k \Rightarrow 24 \mid a_{k+1}$, and also failure to complete the induction argument in a satisfactory way. Of course, this was impossible if $24 \mid \phi_0$ had not been previously established.

Answer: $a_{n+1} - a_n = 24[(12)(17)^{2n} + 9^n]$.

Question 4

This turned out to be a successfully answered question. The majority of candidates produced a complete and correct response.

(i) Not everyone integrated $x\mathrm{e}^{-x^2}$ correctly and moreover there were some errors in the application of the limits.

(ii) Most responses showed correct working to establish the displayed reduction formula. Nevertheless, very few responses showed the optimal strategy, based on a consideration of $D(x^{n+1}\mathrm{e}^{-x^2})$. Instead it was common for candidates to begin by considering the integrand of I_{n+2} as $(x_{n+1})(x\mathrm{e}^{-x^2})$ and then to apply the integration by parts rule to this situation. Some, by a similar argument, obtained I_n in terms of I_{n-2} and then changed n to $n + 2$ so as to prove the required result. This is a satisfactory way to proceed, though it should be emphasised that if *n is* general then so is $n + 2$.

(iii) Surprisingly, there were more errors here than in either of the preceding parts of this question. All that was required was to use $I_3 = I_1 - \frac{1}{2\mathrm{e}}$, $I_5 = 2I_3 - \frac{1}{2\mathrm{e}}$ (common erroneous variants of these equations where $I_3 = 2I_1 - \frac{1}{2\mathrm{e}}$, $I_5 = 3I_3 - \frac{1}{2\mathrm{e}}$) so as to express I_5 in terms of I_1 and then to apply the result obtained in part **(i)**.

Answers: **(i)** $I_1 = \frac{1-\mathrm{e}^{-1}}{2}$; **(iii)** $I_5 = 1 - \frac{5}{2\mathrm{e}}$.

Question 5

There were very few complete and correct responses to this question.

(i) Only about half of all candidates could produce a satisfactory argument to show that $\lim_{\theta \to 0} y = 1$.

Moreover, few understood that in order to show that the line $y = 1$ is an asymptote of C it is also necessary to prove that r or $x \to +\infty$ as $\theta \to 0$, yet only a small number of candidates produced such a proof.

(ii) Again, many candidates were not able to produce a correct sketch of C. Here it was expected of candidates that the line $y = 1$ would appear, but this was frequently omitted. It was also common for the curve to be drawn starting from the pole.

(iii) Almost all responses began with a correct integral representation of the area of the sector OPQ and went on to carry out the integration of θ^{-2} correctly. It was in the completion, which at the purely manipulative level involved only sub A Level mathematics, that some responses fell apart so that again there was evidence of lack of basic mathematical skills.

(iv) This concluding section of the question was very well answered. Most candidates started (correctly) with something like

$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -\theta^{-2} \Rightarrow s = \int_{\pi/6}^{\pi/3} \sqrt{\theta^{-2} + \theta^{-4}}\,\mathrm{d}\theta$ and so went on to obtain the required result.

Answers: **(i)** $y = r\sin\theta = \frac{\sin\theta}{\theta} \to 1$ as $\theta \to 0$, $r = \frac{1}{\theta} \to +\infty$ as $\theta \to 0+$;

(iii) Area of sector $OPQ = \frac{3}{2\pi}$.

Question 6

Most responses showed a good understanding of implicit differentiation and, generally, the working was accurate. Nevertheless, few candidates produced a complete and correct response to this question.

(i) Most responses got as far as exhibiting the preliminary result

$3x^2 + y^2 + 2xy\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + y^2 - 3y^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 0.$

From this the majority of candidates argued (essentially) that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 3x^2 + y^2 = 0$ (*) which is impossible since x and y are necessarily real. This is an incorrect argument since in fact (*) does have the (unique) solution, $x = y = 0$. Thus to complete the argument it is necessary to say that as (0, 0) is not on the curve (this requires formal verification) then $\frac{\mathrm{d}y}{\mathrm{d}x}$ is not zero at any point on it. However, relatively few candidates argued in this way.

(ii) The level of accuracy, both in the further differentiation and in the arithmetic, was generally impressive. The most persistent error was the writing of $D\left(3y^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right)$ as $6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + 3y^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$ and not as $6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 3y^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$.

Answers: **(ii)** At (1, –1), $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{5}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{198}{125}$.

Question 7

This question was generally well answered and the working was, for the most part, accurate.

Strangely, it was in parts **(i)** and **(ii)**, rather than in the later material, that the working sometimes ran unnecessary complications. In part **(i)**, for example, it was common to see $\frac{1}{z} = \frac{1}{\cos\theta + \mathrm{i}\sin\theta} = \frac{\cos\theta - \mathrm{i}\sin\theta}{(\cos\theta - \mathrm{i}\sin\theta)(\cos\theta + \mathrm{i}\sin\theta)} = \frac{\cos\theta - \mathrm{i}\sin\theta}{\cos^2\theta + \sin^2\theta} = \frac{\cos\theta - \mathrm{i}\sin\theta}{1} = \cos\theta - \mathrm{i}\sin\theta$ whereas in its place all that was required was $\frac{1}{z} = (\cos\theta + i\sin\theta)^{-1} = \cos(-\theta) + \mathrm{i}\sin(-\theta)$ (by deMoivre's theorem) $= \cos\theta - \mathrm{i}\sin\theta$ and there were also similar complications in some responses to part **(ii)**.

For the rest, responses developed along correct lines in the way demanded by the question, and the working was generally accurate.

Answers: $a = \frac{15}{16}$, $b = \frac{1}{16}$.

Question 8

This question showed that almost all candidates had a good understanding of how to solve a linear second order differential equation. Nevertheless, there seemed to be a generally limited understanding of the form of the solution obtained, so that relatively few completely correct responses actually appeared.

Almost all candidates obtained the AQE, solved it and formed the correct complementary function. Working for the obtaining of the particular integral was usually accurate so that a correct general solution for the given differential equation was a feature of most scripts.

The majority of candidates understood, in principle, how to apply the given initial conditions and here again most responses led to a correct result for y in terms of t. A small minority of candidates, however, applied the given initial conditions to the complementary function and then added the particular integral in an attempt to find the required solution.

Most candidates comprehended that the complementary function tends to zero as t tends to positive infinity and so concluded (correctly) that $y \approx \sin 3t$ when t is large and positive.

However, from this point onwards, very few made any significant progress. Even the preliminary inequality $\sin 3t < -0.5$ (*) was obtained by only a minority, even though it follows almost immediately from $10^8 y + 10^9 < 9.5 \times 10^8$. The concluding argument would then follow immediately by observing that for $0 \leqslant 3t \leqslant 2\pi$, (*) $\Rightarrow \frac{7\pi}{6} < 3t < \frac{11\pi}{6}$ and that $\frac{\frac{11\pi}{6} - \frac{7\pi}{6}}{2\pi} = \frac{1}{3}$. Nevertheless such reasoning appeared in few scripts even though it required no more than a knowledge of very basic A Level mathematics.

Answer: $y = 4\mathrm{e}^{-2t} - \mathrm{e}^{-3t} + \sin 3t$.

Question 9

The responses to this question showed that the majority of candidates had a sound understanding of the application of scalar and vector products to problems involving 3-dimensional metric vectors. They also showed some understanding of the geometry specific to this question.

Most arguments began with the use of vector products to determine vectors, $\mathbf{n}_1$ and $\mathbf{n}_2$, perpendicular to the planes Π_1 and Π_2 and then went on evaluate $\mathbf{n}_1 \times \mathbf{n}_2$ so as to obtain the vector equation of l. This strategy generated very few errors.

In contrast, a minority of candidates equated the components of the vector equations of Π_1 a[…] expressed three of θ_1, ϕ_1, θ_2, ϕ_2 in terms of the fourth and again this will lead to the requ[…] However, this strategy inevitably generated more errors than the first, and thus showed the clear ad[…] that derives from a having a complete understanding of all the syllabus methodology. This question[…] others like it, become much more formidable if no use can be made of the vector product.

In the middle part of the question, almost all candidates obtained a vector not parallel to *l* (2**i** - **j** - 4**k** was the most popular) in the plane Π_3. Also, they usually went on to obtain both a vector equation of Π_3, often giving it in the form **r.n** = *p*, and a scalar equation of Π_3, though, of course, lack of knowledge of the vector product made this exercise more difficult than it need have been.

In the final part of this question, many candidates argued that as the three given linear equations represent Π_1, Π_2, Π_3, respectively, and as also the line *l* is common to the three planes, then the given system, **S**, has an infinite number of solutions. This is correct as far as it goes, but it also requires that the first two equations be shown to represent Π_1, Π_2, and this vital part of the argument was omitted by some candidates. Of course, in the preceding part of the question, the third equation would have been shown to represent Π_3.

The alternative strategy, adopted by many other candidates, was to reduce the augmented matrix, **A**, to the echelon form and this simple operation was usually carried out accurately. Nevertheless, it was common for the concluding argument to be deficient in some way. Thus to say that as the rank of **A** is equal to 2, then **S** has an infinite number of solutions is incorrect, for **S** might have no solutions. Again, and similar, the fact that the echelon form contains a row of zeros does not of itself imply that **S** has an infinite number of solutions. That depends on the position of the zeros in the rows which are non-zero row vectors. Finally, the still weaker argument that det **B** = 0 (*), where **B** is the matrix of coefficients appertaining to **S**, is clearly false, for (*) is necessary for the conclusion but not sufficient.

Answers: A vector equation of *l* is **r** = 2**i** + 4**j** + 6**k** + *s*(**i** + 2**j** - **k**);
A vector equation of Π_3 is **r** = 2**i** + 4**j** + 6**k** + *s*(**i** + 2**j** - **k**) + *t*(2**i** - **j** - 4**k**);
A scalar equation of Π_3 is $9x - 2y + 5z = 40$.

Question 10

This was the least successful question of the Paper, by quite a long way. Although some responses showed correct strategies for parts **(i)** and **(ii)**, only a minority of all candidates made significant progress with part **(iii)**.

(i) A not uncommon error was the misreading of at least one of the elements of the matrix **H** and, of course, such an inaccuracy undermined much of the subsequent working. It must be emphasised therefore, that in situations, such as this, where there is a lot of numerical data, it is essential that the candidate carries out a thorough check of the copying of this information before becoming involved in the subsequent working. Moreover, there were also a number of arithmetic errors in attempts (not always complete) to obtain a valid echelon form and in some such cases the end product turned out to be a matrix of rank 4. This shows, yet again, the need for continuous checking.

(ii) Most candidates understood that a basis for the null space of *T* could be obtained from three relevant linear equations. In this context, not everyone used the row echelon form they had obtained in part **(i)** from which the required basis can readily be obtained. Instead, a substantial minority used the given form of **H** and so embarked on a more extended strategy. Nevertheless the overall standard of working accuracy was very satisfactory so that most candidates obtained the required basis.

(iii) Here there was a sharp dichotomy of the candidature into the majority who had no idea how to begin and the minority who knew a valid method and applied it in an accurate way. Such a method starts with a consideration of the general solution of the given vector equation, namely

$$\mathbf{x} = \begin{pmatrix} 1 \\ -3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

From this it follows immediately that $|\mathbf{x}|^2 = 1 + (\lambda - 3)^2 + (\lambda - 1)^2 + (\lambda - 2)^2$ and the simple minimisation exercise which in the majority of effective responses was carried ou the above as a quadratic polynomial in λ and then by applying the completion of the technique. Thus the obtaining of another particular solution and then hoping for the be strategy employed by some candidates, is not a valid method.

Among those who used calculus there were some who considered $|\mathbf{x}|$ rather than $|\mathbf{x}|^2$ and who thus became involved in unnecessary complication.

Answers: **(i)** Dimension of range space of *T* is 3; **(ii)** A basis for the null space of *T* is $\begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}$;

(iii) Least possible value of $|\mathbf{x}|$ is $\sqrt{3}$.

Question 11 EITHER

Although less popular than the alternative for **Question 11**, it nonetheless generated much good work. The level of numerical accuracy in responses to the later part of this question was impressive, especially in cases where candidates were involved in unnecessarily complicated strategies. For responses to part **(i)** something like the following was expected. $\mathbf{Ge} = \lambda\mathbf{e} \Rightarrow$, $(\mathbf{G} + k\mathbf{I})\mathbf{e} = \mathbf{Ge} + k\mathbf{Ie} = \lambda\mathbf{e} + k\mathbf{e} = (\lambda + k)\mathbf{e}$, and no comment would then be necessary. Similarly in part **(ii)** the following detail should appear in a complete response. $\mathbf{G}^2\mathbf{e} = \mathbf{G}(\mathbf{Ge}) = \mathbf{G}(\lambda\mathbf{e}) = \lambda(\mathbf{Ge}) = \lambda(\lambda\mathbf{e}) = \lambda^2\mathbf{e}$, and again no comment would be required. However, few candidates produced complete responses to both parts.

For the rest of this question, most responses showed working leading to a correct characteristic equation for the matrix **A** from which the eigenvalues and a set of corresponding eigenvectors were obtained without error. It was also good to see that care was taken to ensure that eigenvalues and eigenvectors were paired off correctly.

Many candidates failed to perceive that $\mathbf{B} = \mathbf{A} - 8\mathbf{I}$ and hence could not exploit the results of parts **(i)** and **(ii)** so as to obtain the eigenvalues and eigenvectors of $\mathbf{B}^2$ in a very simple way. Instead, they attempted to find these scalars and vectors for the matrix **B** and then went on to use **(ii)** to obtain the required results for $\mathbf{B}^2$.

Finally it must be remarked that there was a small subset of candidates who evaluated $\mathbf{B}^2$ and then attempted to find its characteristic equation and so to work on to the final destination. Generally these attempts perished in the large amount of effort required.

Answers: The eigenvalues of **A** are 0, 2, 3.

Corresponding eigenvectors are $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

The above are also eigenvectors of $\mathbf{B}^2$.

The corresponding eigenvalues of $\mathbf{B}^2$ are 64, 36, 25, respectively.

Question 11 OR

In the work for this more popular option of **Question 11** there were many technical deficiencies. Very few completely correct responses appeared.

(i) Usually candidates used an algebraic division process in order to determine *P* and *Q*. However, even this elementary process was not always applied correctly. The simple strategy of putting $x = c$ in $(x - c)(x + P) + Q \equiv (x - a)(x - b)$, so as to obtain *Q* and then considering the coefficient of *x* so as to obtain *P*, was attempted by very few.

(ii) Most responses showed correct equations for the asymptotes and this was often the case when errors had been made in part **(i)**.

(iii) At least half of responses showed $\frac{dy}{dx}$ obtained from the given form of y and not from form exhibited in part **(i)**. This led to a quadratic equation, e.g. $x^2 - 2cx + ac + bc - ab = 0$ hence to the discriminant, Δ, being obtained in the form $4c^2 - 4(ac + bc - ab)$. At this point, who had proceeded along these lines either stated without proof that $\Delta > 0$ and hence that (*) has real roots, etc., or simply gave up and went on to part **(iv)**.

The easier alternative strategy is to start with $\frac{dy}{dx} = 1 - Q(x - c)^{-2} = 0$ from which $x = c \pm \sqrt{Q}$ follows at once. If therefore $Q = (c - a)(c - b)$ had been obtained in part **(i)**, then the given condition $0 < a < b < c$ would show clearly that $Q > 0$ and hence that the roots of (*) are real. However, few candidates produced a complete argument along these lines.

(iv) The main errors here were the drawing of the oblique asymptote so as to cross the positive y-axis, and/or the drawing of the lower branch so as not to intersect the x-axis. Less frequent and less serious errors were bad forms at infinity and failure to indicate in any way the coordinates of the points of intersection of C with the axes. Nevertheless, a substantial number of sketches were complete and correct, though, it must be said, the general quality of diagrams varied substantially across Centres.

Answers: **(i)** $P = c - a - b$, $Q = (c - a)(c - b)$; **(ii)** $x = c$, $y = x + c - a - b$.

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Paper 2

General comments

Almost all candidates attempted all the ten required questions, and some produced excellent work. The overall impression of the Examiners was that candidates performed somewhat better on the compulsory Statistics questions (numbers **5**-**9**) than on the Mechanics ones (**1**-**4**). It was not unusual to see some candidates gaining almost full credit on the former questions, but less so on the latter. The two alternatives for **Question 10** were more challenging, particularly the derivation of the approximate expressions for the tension and cos θ in the mid-part of the Mechanics option, and finding the given values for E(N) and E(N^2) in the Statistics option.

Comments on specific questions

Question 1

Two main alternative approaches were seen to finding the moment of inertia of the lamina, by regarding it as formed from either five squares of side a, or a square of side $3a$ from which the four corner squares have been removed. Less frequently, candidates treated it as composed of a rectangle of sides a and $3a$ together with two small squares. In all cases it was necessary to use the formula for the moment of inertia of the appropriate squares or rectangular lamina about the centre, apply the parallel axes theorem where the centre of the component was other than C, and combine the constituent moments of inertia. One very common error was to take M as the mass of the square of side $3a$ before removal of the four corners. An even more common error in the final part was to use an energy equation with the original moment of inertia about C, instead of the pivot O.

Answer: 6.75 rad s^{-1}.

Question 2

This question is readily answered step-by-step, finding in turn the speed of *P* after it falls 2m, then re

by a factor $\frac{5}{7}$ to give its speed immediately after it hits the floor, and finally its speed after it rises a dista

of 1 m. This latter speed of *P*, together with the speed of *Q* after falling 1 m, are then incorporated into both the conservation of momentum and the restitution equations, and these equations solved to give the required speed of *P* after its collision with *Q*. Those candidates who adopted this methodical approach had little difficulty, other than occasional confusion over signs.

Answers: **(i)** 4.52 m s^{-1}; **(ii)** 3.74 m s^{-1}.

Question 3

Probably the easiest approach is to take moments about the centre, and resolve the forces on the disc both vertically and horizontally. Using the given coefficient of friction to replace either the normal reactions or the limiting friction at *A* and *B* reduces the number of unknowns in these three equations to three, enabling their solution for *P* and for the normal reaction at *A*. Some candidates chose to take moments about *A*, *B* or *C*, which is entirely permissible but somewhat more error-prone.

Answer: $\frac{2W(2-\cos\theta)}{5+3\sin\theta-\cos\theta}$.

Question 4

This seemed to cause more difficulty than any of the other compulsory questions, probably because most candidates did not have a clear idea of the circumstances in which the toboggan might lose contact with the snow. There are two limiting cases to be considered. One is of *U* being too small so that the toboggan has insufficient energy to reach the highest point, and this limiting value is obtained by finding an expression for the velocity at this point and equating it to zero. The second case occurs when *U* is too large so that the normal reaction is zero at some point between *B* and *C*. Expressing this reaction in terms of the normal component of the weight of the toboggan at a general point and the centripetal force $\frac{mv^2}{20}$ and then expressing the velocity *v* in terms of *U*, shows that the critical point when determining the maximum permissible value of *U* is *B* (or *C*). The significance of *B* and *C* is also immediately obvious since *v* and hence the centripetal force are greatest there, while the normal component of the weight is least. The final part of the question involves determining the velocity at *B* of a projectile which has a range of 20 m, and hence the corresponding value of *U* which produces this speed at *B*.

Answer: $10.4 < U < 15.1$; 16.9.

Question 5

Equating the integral of f(x) over [0, 1] to unity in order to find *k* was almost universally done correctly, and most candidates also realised that the starting point in part **(i)** is the distribution function of *X*. Not all, however, appreciated that replacing *x* in it by $\sqrt{y}$ gives the distribution function of *Y*. Differentiation of the latter yields the final result.

Answers: 6; **(i)** $3y - 2y^{3/2}$; **(ii)** $3(1 - y^{1/2})$.

Question 6

This question was usually well-answered, and most candidates rightly chose their equation of *x* on *y* to estimate the English mark corresponding to the History mark of 55. Not all, however, used the square root of the product of the regression coefficients to deduce the product moment correlation coefficient, as implied in the question.

Answer: **(i)** $y = 8.98 + 0.657x$; **(ii)** $x = 18.8 + 0.841y$; 0.743; 65.

Question 7

Having found the expected values for the nine entries in the contingency table, the value 15.85 calculated in the usual way and compared with the tabular value 13.28, leading to the conclusion performance and fuel consumption are not independent. Apart from a small number of candidates who mistakenly combined rows or columns because some of the given data values were smaller than 5, most answers to the main part of the question were completely correct. The two cells in the low/good and high/good position were usually identified correctly, but very few candidates were able to explain the relationship to the context of the question. The explanation preferred by the Examiners is that these cells suggest that good performance is not independent of fuel consumption.

Question 8

Apart from those who essentially used an inappropriate test rather than the two-sample one with equal variances, most candidates were able to calculate a value 2.09 for t and compare it with 2.583, concluding that the mean of doctors' claims is not greater than that of dentists. The confidence interval seemed to be more challenging, with some candidates using a z-value rather than 2.583, or the wrong formula entirely.

Answer: [–4.7, 44.7].

Question 9

Surprisingly many candidates could not estimate p, producing instead the mean number of cracked eggs per box, 0.72, or other less obvious answers. The expected numbers of boxes having each possible number of cracked eggs is then found from the appropriate binomial expansion, and the last five cells combined so that no entry is less than 5. Comparison of the calculated value 0.975 of χ^2 with the tabular value 3.841 leads to the conclusion that a binomial distribution is valid. No doubt because the degree of freedom after combination of the cells is only 1, some candidates applied Yates' correction, but this is only appropriate in the case of a 2×2 contingency table.

Answer: 0.12 .

Question 10

The equilibrium of the particle in the first part of the Mechanics alternative is readily shown by finding from Hooke's Law the vertical components of the tension in the two strings AE and BE, which together equal the particle's weight. The middle parts of the question defeated many candidates, however, who did not realise that they should expand terms such as $(0.25 + 0.6x)^{1/2}$ and $(1 + 1.2x)^{-1}$ and then ignore x^2 and higher powers of x. The approximate period of small oscillations requires the application of Newton's Law for vertical motion, utilising the given approximations for the tensions and cos θ and again neglecting an x^2 term, to obtain an SHM equation and hence the period.

Answer: 0.697 s.

In the Statistics alternative, most candidates were able to write down the values of E(M) and Var(M), but rather fewer could handle the inequality which followed. The attempts at the series for E(M) and E(N) were somewhat disappointing, with some candidates omitting the integers 1, 2, 3, 4, … from successive terms, and even some of those who probably understood how to find the series not giving sufficient terms to fully show their understanding. Relating E(M) and E(N) hinges on one being a constant multiple of the other apart from the first two terms in each, and similarly for the terms in E(M^2) and E(N^2) after the first. Many candidates were, however, able to find Var(N) from the given values of E(N) and E(N^2).

Answers: 4, 12; 0.867; 7.25.