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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01
Paper 1

General comments

This paper was less straightforward overall than its predecessor of a year ago and this difference is reflected in the quality of the work submitted. Thus although the best candidates continued to produce outstanding scripts, the less able found parts of this paper difficult to a significant extent.

There were many attempts to implement unworkable solution strategies and these indicate an excessive dependence on standard procedures. All A Level questions within the framework of this syllabus can be solved expeditiously and thus alternative strategies which involve protracted working should not be implemented. It is important, therefore, that those who prepare for an examination of this type do not fall back entirely on routine examples. Some contact with less routine questions, if not essential, is at least a valuable adjunct to the preparation process.

The majority of candidates made some progress with at least eight questions. Relative to the corresponding examination of November 2002, there was some decline in the quality of presentation of work from the less able candidates. The information about results obtained solely from a graphic calculator given in the rubric was ignored by a significant number of candidates. The basic technical demands of the paper were within the capability of most of the candidature, though, it must be said, elementary arithmetic and algebraic errors were present more than formerly.

The syllabus topics which are calculus-based continued to produce good candidate responses. For the remainder of the topics, complex numbers generated a lot of good work, but summation of series, linear spaces, roots of polynomial equations and 3-dimensional metric vectors showed themselves to be only partially understood. Thus, in particular, responses to **Questions 2**, **3** and **9** were generally inaccurate and incomplete.

Comments on specific questions

Question 1

This introductory question was found to be straightforward by the majority, though nonetheless, there were many elementary errors and/or incorrect strategies to be seen.

The majority of candidates used the correct general formula to obtain for the area $A = \frac{1}{2}\int_0^{\beta} \theta \exp\left(\frac{2\theta^2}{\pi}\right) d\theta$ (*).

In contrast, a small minority wrote $\exp\left(\frac{\theta^4}{\pi^2}\right)$ as the square of $\exp\left(\frac{\theta^2}{\pi}\right)$ and so made no subsequent progress.

The majority of those who obtained (*) were able to carry out the integration accurately and to go on to obtain the required result. Again, however, there was a significant minority who failed simply because they did not recognise that the evaluation of (*) requires little more than the ability to integrate the form $\theta \exp(k\theta^2)$, an exercise that comes well within the scope of basic A Level Mathematics.

Among those, still the majority, who got as far as $A = \frac{\pi}{8}\left[\exp\left(\frac{2\beta^2}{\pi}\right) - 1\right]$, there were few who could not show a convincing argument to obtain the required result for β.

Question 2

There were very few complete and correct responses to this question.

Almost all candidates understood that an application of the difference method was required and went on to obtain a correct result for S_N in terms of N. However, some of the working was confused in that it was unclear what f(n) actually is when writing $u_n = f(n-1) - f(n)$.

In contrast, the second part of this question proved to be the major stumbling block of this paper and only a minority of the candidature made any significant progress here. This general failure was due, in almost all cases, to a supposition that the question, in effect, stated that there was only one possible value of M and that it was up to the candidate to find it. This misconception motivated many candidates to set their result for S_N, usually correct, equal to 10^{-20} and so to become involved in algebraically unrealistic objectives. However, it almost obvious that $S^N < \frac{1}{N^2}$, so that a suitable value of M can easily be obtained.

Answer: $M = 10^{10}$.

Question 3

In general, this question was not well answered. Some candidates appeared not to know the difference between linear dependence and linear independence. Moreover, many failed to see the relevance of the first result to what was required in part **(ii)**.

To begin with, there were a number of different strategies in evidence. The first was an attempt to produce an argument such as $\sum \alpha_i \mathbf{x}_i = \mathbf{0} \Rightarrow \sum \alpha_i \mathbf{M}\mathbf{x}_i = \mathbf{0}$, so that $\mathbf{x}_i$ linearly dependent $\Rightarrow$ not all α_i are zero $\Rightarrow$ $\mathbf{Mx}_i$ are linearly dependent.

However, many candidate responses along these lines were defective in that there was no mention of the requirement that not all α_i are zero.

Other candidates argued that as $\mathbf{x}_i$ linearly dependent $\Rightarrow$ det($\mathbf{x}_1$ $\mathbf{x}_2$ $\mathbf{x}_3$) = 0, then det($\mathbf{Mx}_1$ $\mathbf{Mx}_2$ $\mathbf{Mx}_3$) = det[($\mathbf{M}$($\mathbf{x}_1$ $\mathbf{x}_2$ $\mathbf{x}_3$)] = det($\mathbf{M}$) det($\mathbf{x}_1$ $\mathbf{x}_2$ $\mathbf{x}_3$) = det $\mathbf{M}.\mathbf{0}$ = 0, so that $\mathbf{Mx}_1$, $\mathbf{Mx}_2$, $\mathbf{Mx}_3$ are linearly dependent.

A further strategy, which was attempted by a few, was to argue that $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ linearly dependent $\Rightarrow \exists\, \mathbf{e} \neq 0$ such that ($\mathbf{x}_1$ $\mathbf{x}_2$ $\mathbf{x}_3$) $\mathbf{e} = \mathbf{0} \Rightarrow \mathbf{M}(\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{x}_3)\mathbf{e} = \mathbf{M}.\mathbf{0} = \mathbf{0} \Rightarrow (\mathbf{Mx}_1\ \mathbf{Mx}_2\ \mathbf{Mx}_3)\mathbf{e} = \mathbf{0} \Rightarrow \mathbf{Mx}_1, \mathbf{Mx}_2, \mathbf{Mx}_3$ are linearly dependent.

Such arguments, however, were seldom complete so that as a general conclusion it is clear that most candidates had only a limited understanding of the syllabus material appertaining to this question.

(i) The majority of valid responses showed a linear dependence such as $9\mathbf{y}_1 - 2\mathbf{y}_2 - \mathbf{y}_3 = \mathbf{0}$. An alternative strategy is to show that det($\mathbf{y}_1$ $\mathbf{y}_2$ $\mathbf{y}_3$) = 0 and most working was both accurate and complete in this respect. Likewise the correct reduction of the matrix ($\mathbf{y}_1$ $\mathbf{y}_2$ $\mathbf{y}_3$) to the echelon form so as to establish its rank, followed by a properly argued conclusion, featured in some responses.

(ii) By this stage of the question, it should have been clear to candidates that the basis of the specified linear space must consist of exactly two linearly independent vectors and could therefore be given immediately as {$\mathbf{Py}_1$, $\mathbf{Py}_2$} in numerical form. However, many responses showed the evaluation of {$\mathbf{Py}_1$, $\mathbf{Py}_2$, $\mathbf{Py}_3$}.

Answer: **(ii)** $\left\{ \begin{pmatrix} 2 \\ 45 \\ -49 \end{pmatrix}, \begin{pmatrix} 13 \\ 7 \\ -14 \end{pmatrix} \right\}$.

Question 4

This question was very well answered by the better candidates.

The first part of the question requires no more than routine differentiation and so was well within capability of most candidates.

On the other hand, many responses showed an incorrect inductive hypothesis (IH) and so could not be developed in a valid way. Even where the IH was correctly formulated, there were deficiencies in the inductive proof. Thus with k as the running integer variable, some candidates failed to realise that an increase of 1 in k corresponds to 2 further differentiations. Furthermore, there were confusions of k with n, examples of failure to actually verify that the result is true for $k = 1$ and of differentiation errors. Finally, not all otherwise correct inductive proofs concluded in a satisfactory way in that it was unclear what had actually been proved.

Answer: $\dfrac{d^2y}{dx^2} = 2\cos x - x\sin x$, $\dfrac{d^4y}{dx^4} = x\sin x - 4\cos x$, $\dfrac{d^{2n}y}{dx^{2n}} = (-1)^n(x\sin x - 2n\cos x)$.

Question 5

Most candidates began by establishing the result $\dfrac{d}{dx}[\tan x\sec^n x] = (n+1)\sec^{n+2}x - n\sec^n x$ and from this they were able to go, without difficulty, to the displayed reduction formula. Others ignored the method suggested by the question, and used the integration by parts rule, generally with success. As a matter of detail, it should be pointed out that with the result given, the evaluation of $\tan x\sec^n x$ between 0 and $\dfrac{\pi}{4}$ as $2^{n/2}$ requires the necessary intermediate detail.

The determination of I_6 was effected accurately by most candidates, though surprisingly, not everyone could get a correct result for I_2, or even for I_0.

Answer: $I_6 = \dfrac{28}{15}$.

Question 6

Good responses to the first part of this question were common, but in contrast, there were many failures, both tactical and strategic, in the working for the remainder.

Very few candidates failed to obtain the correct value of the sum of the squares of the roots of the given equation. On the other hand, arguments to show that the given equation has exactly one real root were frequently hazy, to say the least. In this context, it is necessary to highlight the essential aspects of the current mathematical situation, which are:

- since the degree of the given polynomial equation is 3, then it has 3 complex roots
- sum of squares of roots < 0 implies that not all roots are real
- all coefficients of the given polynomial equation are real implies that complex roots occur in conjugate pairs.

The conclusion is then immediate. However, few candidates were able to produce arguments of this clarity and/or completeness.

For the rest of the question, most candidates argued that $f(x) \equiv x^3 + x + 12 \Rightarrow f(-3) = -18$, $f(-2) = 2$ and hence that as $f(-3) < 0$ and $f(-2) > 0$, then $-3 < \alpha < -2$.

They also comprehended that the result $\alpha\beta\beta^* = -12$ (A), which may be obtained directly from the equation, was the key to the obtaining of the final inequality. Generally, this was combined well with (A) so as to obtain the final result. However, a significant minority went wrong at this stage and there were even those who did not recognise that there were 2 inequalities to prove in this question. This suggests that failure to read the question properly induced some reduction in the overall quality of responses.

Answer: Sum of squares of roots of equation = –2.

Question 7

The majority of candidates implemented a sound strategy for obtaining the general solution of second order linear differential equation. However, a minority did not comprehend the significance repeated root of the AQE. Moreover, some wrote the complementary function as $(A + Bx)e^{-2}$ subsequently had 3 variables in the working to obtain the general solution. This suggests that some of th candidature had learnt methods of solution by rote and so had no understanding of what they were trying to do. Suffice to say, solutions of the type of differential equation considered here do not involve extraneous variables.

Some candidates thought that the particular integral was of the form $e^{-\alpha t}$, so completely misinterpreting the significance of α in the context of the question. Such an error, of course, not only precludes the obtaining of the correct general solution, but also makes it impossible to carry out any meaningful attempt to establish the final result.

Many errors appeared in responses to the last part of the question. A minority of candidates produced the form $(A+Bt)e^{(\alpha-2)t}+\dfrac{1}{(2-\alpha)^2}$ for $ye^{\alpha t}$ and even among those who did get this far, there were those who did not spell out an essential preliminary argument such as: $\alpha<2 \Rightarrow \alpha-2<0 \Rightarrow (A+Bt)e^{(\alpha-2)t}\to 0$ as $t\to\infty$, for all constants A and B, before concluding the question as required.

Answer: $y = Ae^{-2t} + Bte^{-2t} + (2-\alpha)^{-2}e^{-\alpha t}$.

Question 8

This question generated a lot of good work and this outcome leads to the conclusion that complex numbers had received serious attention by the majority of those preparing for this examination. It was also good to see that most candidates could take disparate topics on board within the context of a single question.

At the outset, the preliminary results for $z^n \pm \dfrac{1}{z^n}$ were established expeditiously by almost all candidates. Subsequently, the majority used them accurately to establish the required result for $\sin^6\theta$. In a minority of responses, there appeared sign and other elementary arithmetic errors. A few used a 'non – hence' method and so obtained no credit at this stage. Again, therefore, the importance of reading questions accurately must be stressed.

For the final part of the question, most responses showed the correct integral representation of the mean value. At the integration stage, nearly all candidates attempted to make use of the result obtained for $\sin^6\theta$. There were a number of elementary errors in evidence, but nonetheless a correct result was obtained by the majority of candidates.

Answers: $\sin^6\theta = \dfrac{5}{16} - \dfrac{15}{32}\cos 2\theta + \dfrac{3}{16}\cos 4\theta - \dfrac{1}{32}\cos 6\theta$; $\dfrac{5}{16} - \dfrac{11}{12\pi}$.

Question 9

This was a question where choice of optimum methods was essential for complete success within the examination time scale. Regrettably, however, many candidates became involved in suboptimal strategies and so overall did not do at all well with this question.

(i) A simple strategy here is to obtain a vector perpendicular to l_1 and l_2 by means of the vector product and then to find PQ in terms of t by means of the standard formula for the shortest distance between 2 lines. In contrast, many candidates considered R and S, general points on l_1 and l_2, respectively, and so expressed the vector $\overrightarrow{RS}$ in terms of 2 parameters, λ and μ, say. Subsequently, an attempt was made to determine λ and μ in terms of t for $\overrightarrow{RS}$ perpendicular to both l_1 and l_2, i.e. for $R \equiv P$ and $S \equiv Q$ by means of the orthogonality conditions, $(3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}).\ \overrightarrow{RS} = 0$ and $(15\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}).\ \overrightarrow{RS} = 0$.

Such a strategy leads to extremely cumbersome equations and so it was at this stage that most gave up. A few heroic survivors struggled on to the bitter end and they are to be congratulated on their persistence. Nevertheless, it has been remarked in previous Reports on examinations of this syllabus, that it is never the intention of the Examiners to introduce complication for its own sake.

(ii) A popular, but again a less than optimal strategy for the purpose of showing that l t intersect for any t, was to pair off the components of the position vectors of R and S so 3 linear equations in λ and μ and then to go on in an attempt to show inconsistency. Th response was complete in only a minority of cases. A common logical error in this context w argue that if two functions of t are not identical, then they are not equal for any value of t.

Of course, a much more direct way to show that l_1 and l_2 do not intersect is to consider $PQ = \left|\dfrac{20c + 15s - 144}{13}\right|$, where $c = \cos t$ and $s = \sin t$. Since $-25 < 20c + 15s < 25$ (easily proved) then $PQ_{MIN} = \dfrac{119}{13}$. Also, it is clear that $PQ_{MAX} = 13$.

(iii) The most direct strategy here begins by observing that the angle between Π_1 and Π_2 is equal to the angle between l_1 and l_2. As the directions of these lines are given in the data, then the required angle can be evaluated immediately by the standard method. However, only a small minority of candidates argued in this way and this would indicate that the basic geometry of the question was not well understood by the majority.

As it was, the strongly preferred strategy was first to determine the directions of the normals to Π_1 and Π_2 and then to go on to evaluate the angle between them, it being concluded (correctly) that this is also the angle between Π_1 and Π_2. This method not only involves more stages, but also much larger numbers and so is proportionately more error prone. For these reasons there were many elementary errors in evidence. Nevertheless, this part of the question was better answered than parts **(i)** and **(ii)**.

Answers: **(i)** $PQ = \left|\dfrac{20\cos t + 15\sin t - 144}{13}\right|$; **(ii)** 13; **(iii)** 78.2°.

Question 10

This was the best answered question on the paper. Candidates, generally, knew what to do and worked in a well disciplined and efficient way. Almost all failures to obtain correct answers were due to elementary arithmetic errors rather than to incorrect methodology.

An error free characteristic equation appeared in almost all responses. From this, the majority obtained the eigenvalues of **A** and a set of corresponding eigenvectors assembled in the right order. Use was made of the vector product to obtain eigenvectors though sometimes errors occurred in this context.

Responses to the rest of the question were almost always worked accurately by the most direct method. A small minority of candidates attempted, first of all, to express $\mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3$ as a single matrix in numerical form, and then went on to attempt to find the eigenvalues and eigenvectors of this matrix. Such attempts were usually quickly disbanded as it became clear to the candidate that significant progress could not be achieved within the constraints of available examination time.

Answers: Eigenvalues are –1, 2, 8 and a corresponding eigenvectors are $\begin{pmatrix}4\\-9\\8\end{pmatrix}, \begin{pmatrix}5\\-6\\4\end{pmatrix}, \begin{pmatrix}1\\0\\2\end{pmatrix}$,

$\mathbf{P} = \begin{pmatrix}4 & 5 & 1\\-9 & -6 & 0\\8 & 4 & 2\end{pmatrix}$, $\mathbf{D} = \text{diag}(-1, 14, 584)$.

Question 11 EITHER

A majority of the candidature chose this alternative and responses to it were generally satisfact always of high quality.

(i) Almost all candidates obtained correct values for P, Q and R.

(ii) The differentiation was generally correct and the follow on algebra and arithmetic showed up few errors.

(iii) Very few candidates failed to write down the correct equations for the asymptotes. Some got involved in irrelevant and unhelpful preliminary algebra, and this would suggest that the meaning of 'write down' in the context of mathematics examinations is not always understood. In fact, it means that no supporting argument is required, and that therefore only the result gains credit (or not).

(iv) About half of responses showed a sketch graph of y. This was not always consistent with the results established earlier on. A persistent error was the locating of the maximum point on the y-axis. Generally, however, graphs were complete and accurate enough for the purpose of establishing the required range of k. Nevertheless, many stated this to be $1.8 < k < 5$ (A) and this would suggest lack of understanding of what an asymptote actually is.

A substantial minority of responses showed an argument along the lines:

$$\frac{5(x-1)(x+2)}{(x-2)(x+3)} = k \Rightarrow (k-5)x^2 + (k-5)x + 10 - 6k = 0 \, ,$$

so that $x \notin \quad \Leftrightarrow (k-5)^2 - 4(k-5)(10-6k) < 0$, from which the required inequality on k can easily be derived. Such a strategy was generally worked accurately, though again the answer, more often than not, was given as (A), as above.

Answers: **(i)** $P = 5$, $Q = 4$, $R = -4$; **(ii)** $y = 1.8$; **(iii)** $x = 2$, $x = -3$, $y = 5$; **(iv)** $1.8 < k < 5$.

Question 11 OR

The minority who preferred this alternative produced good work for parts **(i)** and **(ii)**, but did markedly less well with part **(iii)**.

(i) Most responses showed the integral representation $s = \int_0^3 (1+x)^{\frac{1}{2}} \mathrm{d}x$ as a starting point and generally this was evaluated accurately. A few used the unnecessary substitution $x = u - 1$ (B), but all the same, this strategy generated few errors.

(ii) Again, the overall strategy for the determination of $\overline{y}$ was generally correct and most evaluations were entirely error-free.

(iii) A significant minority failed to construct a correct integral representation of the area of the surface generated. Thus the incorrect $S = 2\pi \int_0^3 x(1+x)^{\frac{1}{2}} \mathrm{d}y$ appeared in some scripts, as did also $S = 2\pi \int_0^3 y(1+x)^{\frac{1}{2}} \mathrm{d}x$, whereas $S = 2\pi \int_0^3 x(1+x)^{\frac{1}{2}} \mathrm{d}x$ was required.

In this context, the substitution (B), defined above, is certainly helpful and some candidates used it effectively. Others first wrote $S = 2\pi \int_0^3 [(1+x)^{\frac{3}{2}} - (1+x)^{\frac{1}{2}}] \mathrm{d}x$ and yet others used integration by parts, but overall there were many elementary errors.

Answers: **(i)** $\frac{14}{3}$; **(ii)** $\overline{y} = \frac{5\sqrt{3}}{8}$.

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Paper 2

General comments

Virtually all candidates attempted all the eleven required questions, and some produced excellent work. The overall impression of the Examiners was that candidates performed somewhat better on the compulsory Statistics questions (**Questions 6** - **10**) than on the Mechanics ones (**Questions 1** - **5**). Perhaps as a result of this general preference, the Statistics alternative in **Question 11** was almost always chosen instead of the Mechanics one, although a very small number of candidates did produce good answers to the latter.

Comments on specific questions

Question 1

Almost all candidates used the moment of inertia for a rectangular lamina given in the *List of Formulae* to write down the first answer, and most then applied the perpendicular axis theorem correctly to deduce the given moment of inertia about the diagonal *AC*. The last part was rarely answered correctly, however. It requires the realisation that the moment of inertia of the triangular lamina is half that of the square lamina, together with the substitution of $2m$ for M and $\frac{b^2}{8}$ for a^2. One or more of these three operations were frequently omitted, and some candidates attempted to find the moment of inertia from first principles instead of deducing it from the previous result as specified in the question.

Answers: $\frac{2}{3}Ma^2$, $\frac{1}{24}mb^2$.

Question 2

The speed and direction of motion of *E* are readily found by formulating and then solving the conservation of momentum and the restitution equations. Where errors occurred, they were usually due to confusion over signs, best avoided by a clear diagram showing the directions in which the velocities of the trolleys are measured before and after the collision. The magnitude of the force between them is obtained by dividing the impulse on one or other trolley by the contact duration.

Answers: 2 m s^{-1} away from *F*, 90 N.

Question 3

The essential realisation in answering this question is that the tension must be non-negative at the highest point, where the particle is vertically above *Q*. Many candidates, perhaps even the majority, did not consider the highest point, however. Instead they commonly applied conservation of energy to the motion between the lowest point and *Q*, which is unnecessary even as an interim step towards the highest point. It is easier to derive the vertical height between the lowest and highest points and then use conservation of energy to find the speed at the latter. This may then be used to find the tension at the highest point, and hence the required inequality. Another common fault was to require only that the speed rather than the tension should be non-negative throughout the motion.

Question 4

The exact force towards *O* is found by determining the equal tensions in the two strings, and hence their components in the required direction. Expanding $\frac{AP}{2a}$ or something similar in powers of $\left(\frac{x}{2a}\right)^2$ and neglecting terms other than the unit first one produces the given approximation $\frac{\lambda x}{a}$. Showing simple harmonic motion when the two additional strings are introduced requires that the sum of their tensions be added to the approximate force already found, and the result equated to the product of the mass *m* and the particle's acceleration, with careful attention to signs. The period of this motion is then found from the standard SHM formula, which almost all candidates were familiar with though some overlooked *m*.

Answer: $2\pi\sqrt{\frac{ma}{3\lambda}}$.

Question 5

The first given equation follows from equating the net tangential moment to the product of the acceleration and the combined moment of inertia, here 13600 kg m^2. This equation is then integrated the appropriate initial condition, namely $\omega = 0$ when $\theta = \frac{\pi}{3}$, applied. By substituting $\omega = 0$ and $\theta = \pi$ into the equation, the value of F for which the wheel comes to rest may found. Candidates found the final part more challenging, since it requires not only setting the right hand side of the equation found in part **(i)** to zero, but also the selection of an obtuse angle as the resulting value of θ and the evaluation of the corresponding ω using the equation from part **(ii)**.

Answers: **(iii)** 716, 0.488 rad s^{-1}.

Question 6

This question was usually well-answered, with most candidates stating their hypotheses clearly and comparing their calculated t-value of magnitude 2.50 with the tabular value 2.262, concluding that the mean is not 4.58. The comparison was occasionally wrongly made with a critical z-value such as 1.960.

Question 7

This question also presented few difficulties to most candidates, who began by estimating the two-sample common population variance as 20200, and then comparing their calculated t-value of magnitude 1.635 with the tabular value 2.306, concluding that the two methods do not produce concrete with different mean strengths.

Question 8

Having found the mean number of birds alighting per minute, namely 2, and used this to calculate the expected values from a Poisson distribution, most candidates rightly combined the values for 5 or more birds in order that no expected value should be less than 5. Some mistakenly used the value for 7 birds rather than 7 or more birds in the final cell, though this had no significant effect on what followed. The value 5.68 of χ^2 is calculated in the usual way and compared with the tabular value 9.488, leading to the conclusion that a Poisson distribution does indeed fit the data.

Question 9

Despite the first three required answers being given in this question, one or more of these first three parts defeated many candidates, and the final part was only rarely answered satisfactorily. In order to show that the probability that Alec scores first, on his nth kick, is as given, it is essential to explain clearly that this involves both Alec and Bill missing their first $n - 1$ kicks, and then Alec scoring on his nth. While some candidates may have understood this, they failed to give an explicit statement and instead simply wrote down in effect the given expression, thereby gaining no credit. The second and third probabilities may perhaps be demonstrated by a very careful use of conditional probabilities, but very few candidates chose this approach. The easier alternative in part **(ii)** is to state the probability that Alec scores first, on his nth kick, and Bill then misses, and to sum these probabilities for all values of n as a geometric series. Part **(iii)** may be tackled in a broadly similar way, by considering the two alternatives of first Alec and then Bill scoring and also the reverse order. It is also possible to first find the probability that Bill wins the game, which is analogous to part **(ii)** but has numerator $q_1p_2q_1$ rather than p_2q_1, and then subtract the two probabilities of Alec and Bill winning from unity, but this is less straightforward and was not often conducted correctly. Although the final part of the question can be answered by calculating the probability of each number of kicks, and hence summing the appropriate series for the expected value, this was rarely done correctly. A much simpler approach is to consider the game as a succession of kicks with equal probability of scoring, irrespective of which player's turn it is, and then the total number of kicks is one more than the mean 3 for this geometric distribution. Most candidates did not appreciate this, however, and often gave incorrect answers of 3 or 6 without justification, or embarked on lengthy and unsatisfactory calculations.

Answer: 4.

Question 10

Although most candidates integrated f(x) correctly and with the appropriate limits to find P(X < x), m effectively just replaced x by $y^{\frac{1}{3}}$ without valid explanation. A more adequate answer is to note P(Y < y) = P(X^3 < y) = P(X < $y^{\frac{1}{3}}$). Another common fault when finding E(Y) and Var(Y) was to use the given expression for the cumulative distribution function P(Y < y) when integrating rather than the probability density function of Y, found by differentiating the former. Few candidates had any problem with finding the confidence interval, though a satisfactory reason for the interval not to contain the population mean was rarely given. Perhaps the most sensible reason is that there is a 5% chance that the mean lies outside the 95% confidence interval.

Answer: 12.1, 58.6, 10 ± 2.08.

Question 11 EITHER

The first part of this very unpopular alternative question can essentially be answered either by assuming that the friction F_1 is limiting and then using moments and resolutions to arrive at a contradiction, or by showing that the ratio $\frac{F_1}{N_1}$ is less than 0.8 so that F_1 cannot be limiting. If frictions and reactions are related in the second part by, for example, taking moments about H, and replacing N_1 by N_2, then an equation for μ in the limiting case is obtained. The final part requires that three independent equations be obtained for P, Q, N_1, N_2, and then N_1, N_2 eliminated to find $\frac{Q}{P}$.

Answers: **(ii)(a)** 0.75, **(b)** towards A, $\frac{5}{188}$.

Question 11 OR

Although many candidates embarked on this alternative Statistics question by writing down the formulae in terms of summations for the coefficients b in the two regression lines and trying unsuccessfully to solve them, it is easier to treat the two lines as simultaneous equations for the mean coordinates, which turn out to be (13, 6), and hence calculate the missing pair of values. These two steps can of course be combined. Those who found the eighth pair of values correctly and then used the formula given in the *List of Formulae* to calculate the product moment correlation coefficient r usually succeeded, while some of those who instead recalled that r^2 is the product of the given coefficients $-\frac{7}{10}$ and $-\frac{7}{6}$ then unthinkingly chose the positive square root as the value for r. As a comment on the scatter diagram, the Examiners were looking for a statement about the points themselves, such as their lying close to a straight line, rather than simply observing that the correlation coefficient is close to –1. Evaluating the eight values of Y and hence the summation is of course trivial if the missing pair of values has been found correctly, but most candidates were defeated by the second given summation. This sum of the squares of the residuals is a minimum for points lying on the regression line, and so cannot be less in value than 8.8 for any other line.