UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2009 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/02 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu No	Mark Scheme Details			M[illegible]	
1	Find tangential acceleration, $l\,d^2\theta/dt^2$:	$l \cos\theta\, d\theta/dt = l\cos\theta \sin\theta$	M1 A1		
	Find radial acceleration, $l\,(d\theta/dt)^2$:	$l\sin^2\theta$	M1 A1		
	Combine to give $l\,d\theta/dt$ (ignore magnitudes): **A.G.**	$l\sqrt{(\cos^2\theta + \sin^2\theta)}\sin\theta = l\sin\theta$	B1	5	**[5]**
2	Find frequency ω using $T = 2\pi/\omega$:	$\omega = 2\pi/0{\cdot}0225$			
		$[= 800\pi/9 = 279{\cdot}25]$	M1 A1		
	Find v_{max} using $v_{max} = a\omega$:	$v_{max} = 0{\cdot}0105\,\omega\,;\ = 2{\cdot}93[2]$	M1 A1; A1	5	
	Find v using $v^2 = \omega^2(a^2 - x^2)$	$v = \omega\sqrt{(0{\cdot}0105^2 - 0{\cdot}0055^2)}$			
	or $\omega t = \sin^{-1}(x/a) = [0{\cdot}5513]$, $v = a\omega\cos\omega t$:	$[t = 0{\cdot}00197]$, $v = 2{\cdot}50$ **A.G.**	M1 A1	2	**[7]**
3	Use perp. axes theorem for both discs (or lamina):	$I_{2a} = m_{2a}a^2$ *or* $I_a = \frac{1}{4}m_a a^2$	M1		
	Combine to find MI of lamina about diameter (or T):	$I = I_{2a} - I_a$ $[= a^2(m_{2a} - \frac{1}{4}m_a)]$	M1		
	Use par. axes theorem for lamina (or both discs):	$I_T = I + 4ma^2$	M1		
	Find masses of both discs in terms of m:	$m_{2a} = 4m/3$ *and* $m_a = m/3$	B1		
	Combine to find MI of lamina about T:	$I_T = a^2(4 - \frac{1}{4})\,m/3 + 4ma^2$			
		$= 5ma^2/4 + 4ma^2 = 21ma^2/4$ **A.G.**	A1	5	
	Relate initial KE to change in PE at highest pt:	$\frac{1}{2} I_T\,\omega^2$ *and* $4mga$	M1 A1		
	Find set of values [or max. value] of ω (A.E.F.):	$\omega < \sqrt{(32g/21a)}$			
		or $1{\cdot}23\sqrt{(g/a)}$ *or* $3{\cdot}90/\sqrt{a}$	A1	3	**[8]**
4	Find R_A by taking moments about C for system:	$1{\cdot}4R_A = 1{\cdot}0 \times 14$, $R_A = 10$ **A.G.**	M1 A1	2	
	Deduce by taking moments about O for sphere:	$F_B = F_C$ **A.G.**	B1		
	Resolve horizontally for system:	$F_A = F_C$ **A.G.**	B1	2	
	Find any F by e.g. vertical resolution for AB	$F_B = 14 - R_A = 4$	M1 A1		
	or taking moments about B for AB:	$F_A = (0{\cdot}8R_A - 0{\cdot}4 \times 14)/0{\cdot}6 = 4$	(M1 A1)		
	Find R_B by e.g. hor. resolution for rod or sphere:	$R_B = F_A$ *or* F_C $[= 4]$	M1		
	Find R_C by e.g. vert. resolution for sphere or system:	$R_C = 36 + F_B$ *or* $50 - R_A = 40$	M1 A1		
	Find μ_{min}:	$\mu_{min} = \max\{F_A/R_A, F_B/R_B, F_C/R_C\}$	M1		
		$= \max\{4/10, 4/4, 4/40\} = 1$	A1	7	**[11]**

Qu No	Mark Scheme Details					tal
5	(i)	Find max. speed of B using elasticity:	$v = e_1 u \le u$ [*or* $< u$] **A.G.**	M1 A1	2	
	(ii)	Use conservation of momentum:	$m_1 u = m_2 v \le m_2 u$ **A.G.**	M1 A1	2	
		Equate speeds normal to wall, e.g.:	$V \sin \alpha = ev \sin 60°$ *or* $ev\sqrt{3}/2$	M1		
		Equate speeds parallel to wall, e.g.:	$V \cos \alpha = v \cos 60°$ *or* $v/2$	M1		
		Eliminate α by squaring and adding:	$V^2 = v^2 (e^2 \sin^2 60° + \cos^2 60°)$	M1		
		Relate KEs:	$\tfrac{1}{2}mV^2 = \tfrac{1}{3} (\tfrac{1}{2}mv^2)$	B1		
		Hence eliminate speeds to find e:	$e^2 = (\tfrac{1}{3} - \tfrac{1}{4}) / \tfrac{3}{4} = 1/9,\ e = \tfrac{1}{3}$	M1 A1		
		Show that B leaves wall at 30°: **A.G.**	$\tan \alpha = e \tan 60° = 1/\sqrt{3},\ \alpha = 30°$	M1 A1	8	**[12]**
6		Find sample mean:	$\bar{x} = \tfrac{1}{2}(481 + 509) = 495$	M1 A1		
		Use or imply confidence interval formula:	$\bar{x} \pm ts/\sqrt{n}$, any t *or* z [$s = 29{\cdot}9$]	M1		
		Find 90% interval semi-width:	$(t_{19,\,0.95} / t_{19,\,0.975})\ 14$	M1		
		(1·725/2·086 *or* 1·645/1·96 lose A1 only)	= (1·729 / 2·093) 14 = 11·6	A1		
		Hence 90% confidence interval:	[483·4, 506·6] *or* [483, 507]	A1	6	**[6]**
7	(i)	State choice of line with reason (A.E.F.):	y depends on x so choose y on x	B1		
		Find coefficient b in regression line for y:	$b = (66{\cdot}1 - 3{\cdot}25 \times 268/10) / (1{\cdot}2625 - 3{\cdot}25^2/10)$			
			$= -21/0{\cdot}20625 = -101{\cdot}8$ *or* -102	M1 A1		
		Find equation of regression line:	$y = b(x - 0{\cdot}325) + 26{\cdot}8$			
			$= 59{\cdot}9 - 102\,x$	M1 A1		
		SR: M1 A1 for finding x on y:	$x = 0{\cdot}563 - 0{\cdot}00888\,y$	(M1 A1)	5	
	(ii)	Find x when $y = 0$:	59·9/102 = 0·587 *or* 0·588 *or* 0·59	M1 A1		
		SR: If using eqn of x on y:	0·563	(B1)		
		Valid comment on reliability:	OK since point just outside range			
		or	OK as $r \approx -1$ *or* $\lvert r \rvert \approx 1$ (A.E.F.)	B1	3	**[8]**

Qu No	Mark Scheme Details				tal
8	Consider differences $N - A$ (*or* $A - N$):	–10 4 0 –2 –9 1 5 –5 –7	M1		
	Estimate mean and population variance (to 3 sf):	$\overline{x} = -23/9$ [= – 2·556; allow +]			
	and	$s^2 = (301 - 23^2/9) / 8$			
		= 30·28 *or* $5{\cdot}503^2$			
	(allow biased)	(26·91 *or* $5{\cdot}188^2$)	M1 A1		
	Use valid formula (M1 needs n consistent with s):	$\overline{x} \pm ts/\sqrt{n}$ for any t	M1		
	Use correct tabular t value:	$t_{8,\ 0.95}$ = 1·86[0]	*B1		
	Evaluate confidence interval (dep *B1):	– 2·56 ± 3·41 *or* [– 5·97, 0·86]	*A1	6	
	Use valid argument consistent with interval:	0 lies in interval (A.E.F.)	M1		
	State conclusion (dep *A1 apart from rounding):	Yes, supports claim (A.E.F.)	A1	2	**[8]**
9	State (at least) null hypothesis (A.E.F.):	H_0: Sydney popn. has same freq.			
		[0·38, 0·10, 0·03, 0·49]	B1		
	Calculate expected values:	76 20 6 98	M1 A1		
	Calculate value of χ^2 (M1 even if cells combined):	$\chi^2 \approx$ 8·5[2]	M1 *A1		
	Compare with consistent tab. value (to 2 dp):	$\chi_{3,\ 0.95}{}^2$ = 7·815 [$\chi_2{}^2$ = 5·991]	*B1√		
	Correct conclusion (dep *A1, *B1):	Sydney does **not** conform (AEF)	B1	7	
	Find smallest size n with expected values ≥ 5 [or >]:	$n \geq 5/0{\cdot}03 = 166{\cdot}7$ so n_{min} is 167	M1 A1	2	**[9]**

Qu No	Mark Scheme Details				tal
10	Relate $P(T > t)$ to number of hits:	$P(T > t) = P(\text{hits in } t \text{ mins} = 0)$	M1		
	Relate to Poisson distribution:	$= P_0(0{\cdot}8t) = e^{-0{\cdot}8t}$ **A.G.**	M1 A1		
	Find distribution $F(t)$ of T:	$F(t) = P(T < t) = 1 - e^{-0{\cdot}8t}$	B1		
	Differentiate to find $f(t)$ in required form:	$f(t) = 0{\cdot}8\, e^{-0{\cdot}8t}$	B1	5	
	EITHER: State or imply required probability:	$S = \Sigma_{i=1}^{50} T_i$	M1		
	State or use mean of S:	$\mu_S = 50\,(1/0{\cdot}8) = 62{\cdot}5$	A1		
	State or use variance of S:	$\sigma_S^2 = 50\,(1/0{\cdot}8)^2 = 78{\cdot}1[25]$	A1		
	Justify use of Normal approxn. (A.E.F.):	By Central Limit Theorem *or* 50 is large *or* 50 > e.g. 30	B1		
	Evaluate approximate probability (A.E.F):	$1 - \Phi((60 - \mu_S)/\sigma_S)$	M1		
		$= \Phi(0{\cdot}283)$; $= 0{\cdot}611$	A1; A1		
	OR: State or imply required probability:	P(hits in 60 mins < 50) [allow ≤ 50]	(M1)		
	State or use mean:	$\mu = 60 \times 0{\cdot}8 = 48$	(A1)		
	State or use variance:	$\sigma^2 = 60 \times 0{\cdot}8 = 48$	(A1)		
	Justify use of Normal approxn. (A.E.F.):	48 is large *or* 48 > e.g. 15	(B1)		
	Evaluate approximate probability (A.E.F):	$\Phi((49{\cdot}5 - \mu)/\sigma)$	(M1)		
		$= \Phi(0{\cdot}216[5])$; $= 0{\cdot}586$	(A1; A1)	7	
	S.R. Omission of continuity correction:	$\Phi((50 - \mu)/\sigma) = 0{\cdot}614$ earns B1			**[12]**

Qu No	Mark Scheme Details			Part Mark	Total
11 EITHER	Find tension T:	$T = 4mgx/a$	B1		
	Apply Newton's law of motion to B:	$m\,\mathrm{d}^2x/\mathrm{d}t^2 = mg - T$	M1		
	Combine:	$\mathrm{d}^2x/\mathrm{d}t^2 = -(g/a)(4x - a)$ **A.G.**	A1	3	
	Substitute e.g. $y = x - \frac{1}{4}a$ and rearrange:	$\mathrm{d}^2y/\mathrm{d}t^2 = -(4g/a)\,y$	M1 A1		
	State centre of motion, or derive from $y = 0$:	$x_c = \frac{1}{4}a$	A1	3	
	Find x when A starts to slip using $F = \mu R$:	$T = \frac{1}{3}mg,\ x_s = a/12$	M1 A1		
	Valid use of SHM eqn to find time t_s to slipping:	$y = y_{max}\cos\omega t$ *or* $y_{max}\sin\omega t$	M1		
	EITHER: Valid use of cosine form:	$y_s = y_{max}\cos\omega t_s$	M1		
	OR: Valid use of sine form:	$t_s = t_1 - t_2,\ y_{max} = y_{max}\sin\omega t_1$			
		$y_s = y_{max}\sin\omega t_2$	(M1)		
	Substitute for y_s, y_{max} :	$y_s / y_{max} = (x_c - x_s) / x_c$	M1		
		$= (a/6)/(a/4) = \frac{2}{3}$	A1		
	Find t_s :	$t_s = (\cos^{-1}\frac{2}{3})/\omega$			
		or $(\frac{1}{2}\pi - \sin^{-1}\frac{2}{3})/\omega$	M1		
	Substitute $\omega = 2\sqrt{(g/a)}$ and evaluate:	$t_s = 0{\cdot}421\sqrt{(a/g)}$	A1	8	**[14]**
11 OR	State hypotheses:	H_0: $\mu_E = \mu_W$, H_1: $\mu_E \neq \mu_W$	B1	1	
	State assumption [A.E.F.]:	Two populations have Normal distns. and common variance	B1		
	Estimate common variance:	$\sigma^2 = (5 \times 0{\cdot}0231 + 4 \times 0{\cdot}0195)/9$	M1		
		$= 0{\cdot}0215$ *or* $0{\cdot}1473^2$	A1		
	Use correct tabular value of t:	$t_{9,\,0.975} = 2{\cdot}26$[2]	B1		
	Formulate rejection region (with any t; allow >):	$\lvert \bar{x}_E - \bar{x}_W \rvert \geq t\sigma\sqrt{(1/6 + 1/5)}$	M1		
		$= 0{\cdot}201$	A1		
	Compare actual sample means with region:	$0{\cdot}253 > 0{\cdot}201$√			
	or compare calculated t with tabular t:	$2{\cdot}85 > 2{\cdot}26$[2]√	M1 A1		
	Consistent conclusion (A.E.F.; dep values above):	Mean acidity levels **do** differ	A1	9	
	State condition on a (with any t; allow > or =):	$\bar{x}_E - \bar{x}_W - a \geq t\sigma\sqrt{(1/6 + 1/5)}$	M1		
	Use correct tabular value of t:	$t_{9,\,0.95} = 1{\cdot}83$[3]	A1		
	Substitute to find largest value of a:	$a_{max} = 0{\cdot}253 - 0{\cdot}163 = 0{\cdot}09$ (2 dp)	M1 A1	4	**[14]**