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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2010**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

4278730524

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

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1 Given that 5 is an eigenvalue of the matrix

$$\mathbf{A} = \begin{pmatrix} 5 & -3 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix},$$

find a corresponding eigenvector. [2]

Hence find an eigenvalue and a corresponding eigenvector of the matrix $\mathbf{A} + \mathbf{A}^2$. [2]

2 By considering the identity

$$\cos[(2n-1)\alpha] - \cos[(2n+1)\alpha] \equiv 2\sin\alpha \sin 2n\alpha,$$

show that if α is not an integer multiple of π then

$$\sum_{n=1}^{N} \sin(2n\alpha) = \tfrac{1}{2}\cot\alpha - \tfrac{1}{2}\operatorname{cosec}\alpha \cos[(2N+1)\alpha].$$ [4]

Deduce that the infinite series

$$\sum_{n=1}^{\infty} \sin\left(\tfrac{2}{3}n\pi\right)$$

does not converge. [1]

3 The sequence $x_1, x_2, x_3, \ldots$ is such that $x_1 = 3$ and

$$x_{n+1} = \frac{2x_n^2 + 4x_n - 2}{2x_n + 3}$$

for $n = 1, 2, 3, \ldots$. Prove by induction that $x_n > 2$ for all n. [6]

4 The parametric equations of a curve are

$$x = \cos t + t\sin t, \qquad y = \sin t - t\cos t.$$

The arc of the curve joining the point where $t = 0$ to the point where $t = \tfrac{1}{2}\pi$ is rotated about the x-axis through one complete revolution. Find the area of the surface generated, leaving your result in terms of π. [7]

5 Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$$ [4]

Hence find all the roots of the equation

$$32x^5 - 40x^3 + 10x + 1 = 0$$

in the form $\sin(q\pi)$, where q is a positive rational number. [4]

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6 The curve C has equation

$$y = \frac{x^2 - 3x - 7}{x + 1}.$$

(i) Obtain the equations of the asymptotes of C. [3]

(ii) Show that $\frac{dy}{dx} > 1$ at all points of C. [2]

(iii) Draw a sketch of C. [3]

7 It is given that

$$x = t^2 e^{-t^2} \quad \text{and} \quad y = t e^{-t^2}.$$

(i) Show that

$$\frac{dy}{dx} = \frac{1 - 2t^2}{2t - 2t^3}.$$ [3]

(ii) Find $\frac{d^2y}{dx^2}$ in terms of t. [5]

8 Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 10 \sin 3x - 20 \cos 3x.$$ [5]

Show that, for large positive x and independently of the initial conditions,

$$y \approx R \sin(3x + \phi),$$

where the constants R and ϕ, such that $R > 0$ and $0 < \phi < 2\pi$, are to be determined correct to 2 decimal places. [4]

9 Let

$$I_n = \int_0^{\frac{1}{2}\pi} \sin^n \theta \, d\theta,$$

where n is a non-negative integer. Show that $I_{n+2} = \frac{n+1}{n+2} I_n$. [4]

The region R of the x-y plane is bounded by the x-axis, the line $x = \frac{\pi}{2m}$ and the curve whose equation is $y = \sin^4 mx$, where $m > 0$. Find the y-coordinate of the centroid of R. [6]

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10 The equation

$$x^4 + x^3 + cx^2 + 4x - 2 = 0,$$

where c is a constant, has roots α, β, γ, δ.

(i) Use the substitution $y = \dfrac{1}{x}$ to find an equation which has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}, \dfrac{1}{\delta}$. [2]

(ii) Find, in terms of c, the values of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ and $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2} + \dfrac{1}{\delta^2}$. [3]

(iii) Hence find

$$\left(\alpha - \frac{1}{\alpha}\right)^2 + \left(\beta - \frac{1}{\beta}\right)^2 + \left(\gamma - \frac{1}{\gamma}\right)^2 + \left(\delta - \frac{1}{\delta}\right)^2$$

in terms of c. [2]

(iv) Deduce that when $c = -3$ the roots of the given equation are not all real. [3]

11 The curve C has polar equation

$$r = \frac{a}{1 + \theta},$$

where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Show that r decreases as θ increases. [2]

(ii) The point P of C is further from the initial line than any other point of C. Show that, at P,

$$\tan\theta = 1 + \theta,$$

and verify that this equation has a root between 1.1 and 1.2. [4]

(iii) Draw a sketch of C. [3]

(iv) Find the area of the region bounded by the initial line, the line $\theta = \frac{1}{2}\pi$ and C, leaving your answer in terms of π and a. [3]

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12 Answer only **one** of the following two alternatives.

EITHER

The line l_1 passes through the point A whose position vector is $3\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and is parallel to the vector $\mathbf{i}+\mathbf{j}$. The line l_2 passes through the point B whose position vector is $-\mathbf{i}-\mathbf{k}$ and is parallel to the vector $\mathbf{j}+2\mathbf{k}$. The point P is on l_1 and the point Q is on l_2 and PQ is perpendicular to both l_1 and l_2.

(i) Find the length of PQ. [4]

(ii) Find the position vector of Q. [5]

(iii) Show that the perpendicular distance from Q to the plane containing AB and the line l_1 is $\sqrt{3}$. [4]

OR

The linear transformation $\mathrm{T}: \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & 1 & 5 & 7 \\ 3 & 9 & 17 & 25 \\ 1 & 7 & 7 & 11 \\ 3 & 6 & 16 & 23 \end{pmatrix}$.

(i) In either order,

(a) show that the dimension of R, the range space of T, is equal to 2,

(b) obtain a basis for R.

[5]

(ii) Show that the vector $\begin{pmatrix} 1 \\ -15 \\ -17 \\ -6 \end{pmatrix}$ belongs to R. [3]

(iii) It is given that $\{\mathbf{e}_1, \mathbf{e}_2\}$ is a basis for the null space of T, where $\mathbf{e}_1 = \begin{pmatrix} 14 \\ 1 \\ -3 \\ 0 \end{pmatrix}$ and $\mathbf{e}_2 = \begin{pmatrix} 19 \\ 2 \\ 0 \\ -3 \end{pmatrix}$.

Show that, for all λ and μ,

$$\mathbf{x} = \begin{pmatrix} 4 \\ -3 \\ 0 \\ 0 \end{pmatrix} + \lambda\mathbf{e}_1 + \mu\mathbf{e}_2$$

is a solution of

$$\mathbf{M}\mathbf{x} = \begin{pmatrix} 1 \\ -15 \\ -17 \\ -6 \end{pmatrix}. \qquad (*)$$ [3]

(iv) Hence find a solution of $(*)$ of the form $\begin{pmatrix} \alpha \\ 0 \\ \gamma \\ \delta \end{pmatrix}$. [2]

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