UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2010 question paper for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/02 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details				Total
1	Find period T using $v^2 = \omega^2 (A^2 - x^2)$ and $T = 2\pi/\omega$:	$\omega = 6/4,\ T = 4\pi/3$ *or* $4{\cdot}19$ [s]	M1 A1	2	
	(i) Find max speed using $v_{max} = \omega A$:	$v_{max} = 15/2$ *or* $7{\cdot}5$ [ms^{-1}]	M1 A1	2	
	(ii) Find mag. of max accel. using $a_{max} = \omega^2 A$:	$a_{max} = 45/4$ *or* $11{\cdot}2[5]$ [ms^{-2}]	M1 A1	2	**[6]**
2	Apply conservation of energy:	$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 - mga(1 - \cos\theta)$	M1		
	Put $v = \tfrac{1}{2}u$ and simplify:	$8ga(1 - \cos\theta) = 3u^2$ **A.G.**	A1	2	
	Equate radial forces to find contact force N:	$N = mg\cos\theta + m(\tfrac{1}{2}u)^2/a$	M1 A1		
	Replace $\cos\theta$ by $1 - 3u^2/8ga$ (A.E.F.):	$N = mg - mu^2/8a$	M1 A1	4	**[6]**
3	Use conservation of momentum:	$mu + \tfrac{1}{4}\alpha mu = \alpha m v_B$	M1 A1		
	Use Newton's law of restitution:	$-v_B = -e(u - \tfrac{1}{4}u)$ [$v_B = \tfrac{3}{4}eu$]	M1 A1		
	Eliminate v_B to find e (A.E.F.):	$e = (1 + \tfrac{1}{4}\alpha)/\tfrac{3}{4}\alpha$ *or* $(4 + \alpha)/3\alpha$	M1 A1	6	
	Use $e \le 1$ to find inequality for α:	$4 + \alpha \le 3\alpha$ so $\alpha \ge 2$ **A.G.**	M1 A1	2	**[8]**
4	Resolve in any two dirns. for rod, e.g. vertically:	$R_A \sin 2\theta + R_B \cos\theta = W$			
	or horizontally:	$R_A \cos 2\theta - R_B \sin\theta = 0$			
	or parallel to rod:	$R_A \cos\theta = W\sin\theta$			
	or normal to rod:	$R_A \sin\theta + R_B = W\cos\theta$	B1 B1	2	
	(i) Solve to find R_A, e.g.:	$R_A = W\sin\theta\ /$ $(\cos 2\theta\cos\theta - \sin 2\theta\sin\theta)$ $= W\tan\theta$ **A.G.**	M1 A1	2	
	(ii) Solve to find R_B, e.g.:	$R_B = W\tan\theta\cos 2\theta / \sin\theta$ $= W\cos 2\theta / \cos\theta$ **A.G.**	M1 A1	2	
	(iii) Take moments for rod, e.g. about A: or about B:	$R_B\ 2r\cos\theta = W a\cos\theta$ $R_A\ 2r\cos\theta\sin\theta =$ $W(2r\cos\theta - a)\cos\theta$	M1 A1		
	Substitute and simplify:	$2r\cos 2\theta = a\cos\theta$ **A.G.**	A1	3	**[9]**
5	Find MI of disc about axis at A by par. axes thm:	$I_{disc} = \tfrac{1}{4}\,2ma^2 + 2ma^2$ [$= 5ma^2/2$]	M1 A1		
	Find MI of particle about axis at A:	$I_m = m(2a)^2$ [$= 4ma^2$]	B1		
	Combine to find MI of system:	$I = 13ma^2/2$ **A.G.**	A1	4	
	Use conservation of energy (lose A1 for one error):	$\tfrac{1}{2}I\Omega^2 = 2mg \times a + mg \times 2a$	M1 A2		
	Substitute for I to find angular speed Ω:	$\Omega = \sqrt{(16g/13a)}$ A.E.F.	M1 A1	5	
	State eqn of motion (A.E.F.):	$I\,d^2\theta/dt^2 = -4mga\sin\theta$	M1 A1		
	Approximate $\sin\theta$ by θ (implied by use of SHM):	$I\,d^2\theta/dt^2 = -4mga\,\theta$	M1		
	Find approx. period T from SHM formula:	$T = 2\pi/\sqrt{(8mga/13ma^2)}$ $= 2\pi\sqrt{(13a/8g)}$ A.E.F.	M1 A1	5	**[14]**
6	Use valid formula for C.I.:	$\overline{x}_B - \overline{x}_A \pm z\sigma\sqrt{(1/n_A + 1/n_B)}$	M2		
		$= 112 - 109 \pm z\ 15\sqrt{(1/15 + 1/20)}$	A1		
		$= 3 \pm 5{\cdot}123\,z$	A1		
	Use of correct tabular value:	$z_{0.995} = 1{\cdot}64[5]$	*A1		
	C.I. correct to 3 s.f. (dep *A1):	$3 \pm 8{\cdot}43$ *or* $[-5{\cdot}43, 11{\cdot}4]$	A1	6	**[6]**
7	**(i)** Find or imply value of p:	$p = \tfrac{1}{4}$ *or* $0{\cdot}25$	B1		
	Find $P(X = 5)$:	$(1 - p)^4 p$ *or* $q^4 p = 0{\cdot}0791$	M1 A1	3	
	(ii) Find $P(X \ge 5)$:	$1 - (1 + q + q^2 + q^3)p$ *or* q^4 *or* $q^4 p + q^5 = 0{\cdot}316$	M1 A1	2	
	(iii) Find least N with $P(X \le N) > 0{\cdot}9995$:	$1 - q^N > 0{\cdot}9995,\ q^N < 0{\cdot}0005$			

Q					
8	Find expected values to (at least) 1 dp:	$\quad$ A $\quad$ B $\quad$ C			
	(lose A1 if one or more errors	*Passes* $\quad 18{\cdot}33 \quad 14{\cdot}67 \quad 22{\cdot}00$			
	or if rounded to integers)	*Failures* $\quad 31{\cdot}67 \quad 25{\cdot}33 \quad 38{\cdot}00$	M1 A1		
	State (at least) null hypothesis (A.E.F.):	H_0: Test result indep of school	B1		
	Calculate value of χ^2 :	$\chi^2 = 3{\cdot}7 \pm 0{\cdot}02$	B1		
	S.R. If rounded to integers above allow:	$\chi^2 = 3{\cdot}96$ *or* $4{\cdot}0$ (earns max 6/7)	(B1)		
	Compare with tabular value (to 2 dp):	$\chi_{2,\,0.95}{}^2 = 5{\cdot}99$	B1		
	Valid method for reaching conclusion:	Reject H_0 if χ^2 > tabular value	M1		
	Correct conclusion (A.E.F., requires correct values):	No association	A1	7	[7]
9	State both hypotheses (A.E.F.):	H_0: $\mu_I - \mu_O = 0{\cdot}1$, H_1: $\mu_I - \mu_O > 0{\cdot}1$	B1		
	State valid assumption for paired-sample test:	Popln. of diffs. has Normal distn.	B1		
	Consider differences eg:	$0{\cdot}4 \;\; 0{\cdot}1 \;\; 0{\cdot}2 \;\; 0{\cdot}3 \;\; 0{\cdot}1 \;\; 0{\cdot}4 \;\; 0{\cdot}1 \;\; 0{\cdot}2$	M1		
	Calculate sample mean:	$\overline{d} = 1{\cdot}8 / 8$ [= $0{\cdot}225$]	M1		
	Estimate population variance:	$s^2 = (0{\cdot}52 - 1{\cdot}8^2/8) / 7$			
	(allow biased: $0{\cdot}0144$ *or* $0{\cdot}120^2$)	[= $0{\cdot}0164$ *or* $0{\cdot}128^2$]	M1		
	Calculate value of t (to 2 dp):	$t = (\overline{d} - 0{\cdot}1) / s\sqrt{(1/8)} = 2{\cdot}76$	M1 *A1		
	Compare with correct tabular t value:	$t_{7,\,0.975} = 2{\cdot}36[5]$	*B1		
	Valid method for reaching conclusion:	Reject H_0 if χ^2 > tabular value	M1		
	Correct conclusion (AEF, dep *A1, *B1):	Coach's suspicion is correct	A1	10	[10]
	S.R.: State both hypotheses:	H_0: $\mu_I - \mu_O = 0{\cdot}1$, H_1: $\mu_I - \mu_O > 0{\cdot}1$	(B1)		
	State valid assumption for 2-sample test:	Both poplns. have Normal distns. *and* a common variance	(B1)		
	Calculate sample means:	$170{\cdot}4/8$, $168{\cdot}6/8$ [= $21{\cdot}3$, $21{\cdot}075$]			
	and estimate population variance:	$s^2 = (3630{\cdot}1 - 170{\cdot}4^2/8 + 3553{\cdot}94 - 168{\cdot}6^2/8)/14$ [= $0{\cdot}09107$]	(M1)		
	Calculate value of t (to 2 dp):	$(0{\cdot}225 - 0{\cdot}1)/s\sqrt{(1/8 + 1/8)} = 0{\cdot}828$	(M1*A1)		
	Compare with correct tabular t value:	$t_{14,\,0.975} = 2{\cdot}14[5]$	(*B1)		
	Correct conclusion (AEF, dep *A1, *B1):	Coach's suspicion is not correct	(B1 max 7)		
10	**(i)** Find mean values to 3 s.f.:	$\overline{x} = 2{\cdot}024$, $\overline{y} = 3{\cdot}817$	B1	1	
	(ii) Calculate gradient b in $y - \overline{y} = b(x - \overline{x})$:				
		$b = (88{\cdot}415 - 24{\cdot}29 \times 45{\cdot}8/12) / (50{\cdot}146 - 24{\cdot}29^2/12)$	M1		
		$= -4{\cdot}292 / 0{\cdot}979$			
		or $-0{\cdot}358 / 0{\cdot}0816 = -4{\cdot}38[4]$	A1	2	
	(iii) Find regression line:	$y - 3{\cdot}817 = -4{\cdot}384\,(x - 2{\cdot}024)$	M1		
		$y = 12{\cdot}7 - 4{\cdot}38x$	A1	2	
	(iv) Find correlation coefficient r:				
		$r = (88{\cdot}415 - 24{\cdot}29 \times 45{\cdot}8/12) / \sqrt{\{(50{\cdot}146 - 24{\cdot}29^2/12)\,(211{\cdot}16 - 45{\cdot}8^2/12)\}}$	M1		
		$= -4{\cdot}292 / \sqrt{(0{\cdot}979 \times 36{\cdot}36)}$	A1		
		or $-0{\cdot}358 / \sqrt{(0{\cdot}0816 \times 3{\cdot}03)}$			
		$= -0{\cdot}719$	A1		
	State valid comment in context (A.E.F.):	[Moderate] negative correlation between rainfall and sunshine	A1	4	
	(v) State both hypotheses:	H_0: $\rho = 0$, H_1: $\rho < 0$	B1		
	Use correct tabular r value:	$r_{12,\,1\%} = 0{\cdot}658$	B1		
	Valid method for reaching conclusion:	Reject H_0 if $\lvert r \rvert$ > tabular value	M1		
	S.R. Calculate t-value:	$t = r\sqrt{10} / \sqrt{(1 - r^2)} = -3{\cdot}27$	(B1)		
	Use correct tabular t value:	$t_{10,\,0.99} = 2{\cdot}76[4]$	(B1)		
	Correct conclusion (needs values correct):	There is negative corrln. (A.E.F.)	A1	4	[13]

11 EITHER	Equate tensions at O (A1 for each):	$60\,(1 + MO)/2 \;=\; 20\,(2 - MO)/1$	M1 A1 A1		
	Simplify to evaluate MO:	$MO \;=\; 10/50 \;=\; 0{\cdot}2$ **A.G.**	A1		
	Apply Newton's law at general point:	$0{\cdot}1\ \mathrm{d}^2y/\mathrm{d}t^2 \;=$			
	(lose A1 for each incorrect term)	$20\,(1{\cdot}8 - y) - 60\,(1{\cdot}2 + y)/2$	M1 A2		
	Simplify:	$\mathrm{d}^2y/\mathrm{d}t^2 \;=\; -\,500y$ **A.G.**	A1		
	State period (A.E.F.):	$T \;=\; 2\pi/\sqrt{500}$ *or* $\pi\,/5\sqrt{5}$ *or* $0{\cdot}281$ [s]	B1	5	
	(i) Find speed v when first at 0·3 from M:	$v^2 \;=\; 500\,(0{\cdot}2^2 - 0{\cdot}1^2)$	M1		
		$v \;=\; \sqrt{15}$ *or* $3{\cdot}87$ [m s^{-1}]	A1	2	
	(ii) Find time t to reach this point: (A.E.F.)	$t \;=\; (1/\omega)\cos^{-1}(-0{\cdot}1/0{\cdot}2)$ *or* $\tfrac{1}{4}T + (1/\omega)\sin^{-1}(0{\cdot}1/0{\cdot}2)$	M1		
		$=\; (2\pi/3)\,/\omega$ *or* $(\pi/2 + \pi/6)\,/\omega$	A1		
		$=\; 2{\cdot}094/\sqrt{500}$ [*or* $= 0{\cdot}07025 + 0{\cdot}02342$]			
		$=\; 0{\cdot}0937$ [s]	A1	3	**[14]**
11 OR	Integrate f(t) to find F(t):	$\mathrm{F}(t) \;=\; \int_0^t \lambda e^{-\lambda x}\,\mathrm{d}x \;=\; [-\,e^{-\lambda x}]_0^{\,t}$			
		$=\; 1 - e^{-\lambda t}$ **A.G.**	M1 A1	2	
	EITHER: Deduce λ directly from mean:	$\lambda \;=\; 1/20$ *or* $0{\cdot}05$			
	OR: Deduce λ from a tabular value, e.g.:	$1 - e^{-40\lambda} \;=\; 0{\cdot}8647,\ \lambda \;=\; 0{\cdot}05$	M1		
	Substitute for λ and put $t = 15$ to give F(15) to 4 dp:	$1 - e^{-15/20} \;=\; 0{\cdot}5276$ [*or* 0·5277]	A1	2	
	Calculate expected values to 2 dp (5 values earn A1):	22·12 17·23 13·41 10.45 8.14 6.34 4.93 3·85 13·53	M1 A2	3	
	State (at least) null hypothesis:	H_0: $1 - e^{-t/20}$ fits data (A.E.F.)	B1		
	Combine two adjacent cells with exp. value < 5:	O: . . . 8 6 17			
		E: . . . 6·34 8·78 13·53	M1		
	Calculate value of χ^2 (to 2 dp):	$\chi^2 \;=\; 3{\cdot}58$	M1 A1		
	(Cells not combined gives 4·81 earning M1 A0, max 4/7)				
	Compare with consistent tabular value (to 2 dp):	$\chi_{7,\,0.95}^{\ 2} \;=\; 14{\cdot}07$ (cells combined)			
		$\chi_{8,\,0.95}^{\ 2} \;=\; 15{\cdot}51$ (not combined)	B1		
	Valid method for reaching conclusion:	Reject H_0 if χ^2 > tabular value	M1		
	Correct conclusion (A.E.F., requires correct values):	3·58 < 14·07 so suitable model	A1	7	**[14]**