UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/11**

Paper 1 **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

0372465191

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Express $\dfrac{1}{(2r+1)(2r+3)}$ in partial fractions and hence use the method of differences to f

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)}.$$

Deduce the value of

$$\sum_{r=1}^{\infty} \frac{1}{(2r+1)(2r+3)}.$$ [1]

2 The roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are $\dfrac{\beta}{k}$, β, $k\beta$, where p, q, r, k and β are non-zero real constants. Show that $\beta = -\dfrac{q}{p}$. [4]

Deduce that $rp^3 = q^3$. [2]

3 The linear transformation $\mathrm{T} : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & 3 & -2 & 4 \\ 5 & 15 & -9 & 19 \\ -2 & -6 & 3 & -7 \\ 3 & 9 & -5 & 11 \end{pmatrix}$.

(i) Find the rank of $\mathbf{M}$. [3]

(ii) Obtain a basis for the null space of T. [3]

4 It is given that $\mathrm{f}(n) = 3^{3n} + 6^{n-1}$.

(i) Show that $\mathrm{f}(n+1) + \mathrm{f}(n) = 28(3^{3n}) + 7(6^{n-1})$. [2]

(ii) Hence, or otherwise, prove by mathematical induction that $\mathrm{f}(n)$ is divisible by 7 for every positive integer n. [4]

5 The curve C has polar equation $r = 2\cos 2\theta$. Sketch the curve for $0 \leqslant \theta < 2\pi$. [4]

Find the exact area of one loop of the curve. [4]

6 The line l_1 passes through the point with position vector $8\mathbf{i} + 8\mathbf{j} - 7\mathbf{k}$ and is parallel to the vector $4\mathbf{i} + 3\mathbf{j}$. The line l_2 passes through the point with position vector $7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and is parallel to the vector $4\mathbf{i} - \mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. In either order,

(i) show that $PQ = 13$,

(ii) find the position vectors of P and Q.

[9]

7 The variables x and y are related by the differential equation

$$y^2\frac{d^2y}{dx^2} + 2y^2\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right)^2 - 5y^3 = 8e^{-x}.$$

Given that $v = y^3$, show that

$$\frac{d^2v}{dx^2} + 2\frac{dv}{dx} - 15v = 24e^{-x}.$$ [4]

Hence find the general solution for y in terms of x. [7]

8 Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{A} = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 0 & -3 \\ 1 & -3 & 0 \end{pmatrix}$. [8]

Find a non-singular matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [3]

9 The curve C has equation $y = x^{\frac{3}{2}}$. Find the coordinates of the centroid of the region bounded by C, the lines $x = 1$, $x = 4$ and the x-axis. [7]

Show that the length of the arc of C from the point where $x = 5$ to the point where $x = 28$ is 139. [5]

10 Let

$$I_n = \int_0^{\frac{1}{2}\pi} \cos^n x \, dx,$$

where $n \geqslant 0$. Show that, for all $n \geqslant 2$,

$$I_n = \frac{n-1}{n} I_{n-2}.$$ [4]

A curve has parametric equations $x = a\sin^3 t$ and $y = a\cos^3 t$, where a is a constant and $0 \leqslant t \leqslant \frac{1}{2}\pi$. Show that the mean value m of y over the interval $0 \leqslant x \leqslant a$ is given by

$$m = 3a\int_0^{\frac{1}{2}\pi} (\cos^4 t - \cos^6 t)\, dt.$$ [4]

Find the exact value of m, in terms of a. [4]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

Use de Moivre's theorem to prove that

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$ [6]

State the exact values of θ, between 0 and π, that satisfy $\tan 3\theta = 1$. [2]

Express each root of the equation $t^3 - 3t^2 - 3t + 1 = 0$ in the form $\tan(k\pi)$, where k is a positive rational number. [3]

For each of these values of k, find the exact value of $\tan(k\pi)$. [3]

OR

The curve C has equation

$$y = \frac{x^2 + \lambda x - 6\lambda^2}{x + 3},$$

where λ is a constant such that $\lambda \neq 1$ and $\lambda \neq -\frac{3}{2}$.

(i) Find $\frac{dy}{dx}$ and deduce that if C has two stationary points then $-\frac{3}{2} < \lambda < 1$. [5]

(ii) Find the equations of the asymptotes of C. [3]

(iii) Draw a sketch of C for the case $0 < \lambda < 1$. [3]

(iv) Draw a sketch of C for the case $\lambda > 3$. [3]