UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Find the sum of the first n terms of the series

$$\frac{1}{1 \times 3} + \frac{1}{2 \times 4} + \frac{1}{3 \times 5} + \dots$$

and deduce the sum to infinity. [3]

2 For the sequence $u_1, u_2, u_3, \dots$, it is given that $u_1 = 1$ and $u_{r+1} = \frac{3u_r - 2}{4}$ for all r. Prove by mathematical induction that $u_n = 4\left(\frac{3}{4}\right)^n - 2$, for all positive integers n. [5]

3 The curve C has equation

$$xy + (x + y)^3 = 1.$$

Show that $\frac{dy}{dx} = -\frac{3}{4}$ at the point A (1, 0) on C. [3]

Find the value of $\frac{d^2y}{dx^2}$ at A. [5]

4 Let

$$I_n = \int_1^e x^2 (\ln x)^n \, dx,$$

for $n \geqslant 0$. Show that, for all $n \geqslant 1$,

$$I_n = \tfrac{1}{3}e^3 - \tfrac{1}{3}nI_{n-1}.$$ [4]

Find the exact value of I_3. [4]

5 The matrix $\mathbf{A}$ has an eigenvalue λ with corresponding eigenvector $\mathbf{e}$. Prove that the matrix $(\mathbf{A} + k\mathbf{I})$, where k is a real constant and $\mathbf{I}$ is the identity matrix, has an eigenvalue $(\lambda + k)$ with corresponding eigenvector $\mathbf{e}$. [2]

The matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix}.$$

Two of the eigenvalues of $\mathbf{B}$ are -3 and 4. Find corresponding eigenvectors. [3]

Given that $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ is an eigenvector of $\mathbf{B}$, find the corresponding eigenvalue. [1]

Hence find the eigenvalues of $\mathbf{C}$, where

$$\mathbf{C} = \begin{pmatrix} -1 & 2 & -3 \\ 2 & -1 & 3 \\ -3 & 3 & 0 \end{pmatrix},$$

and state corresponding eigenvectors. [3]

6 The curve C has equation $y = \dfrac{x^2}{x-2}$. Find the equations of the asymptotes of C.

Find the coordinates of the turning points on C.

Draw a sketch of C. [3]

7 Expand $\left(z + \dfrac{1}{z}\right)^4 \left(z - \dfrac{1}{z}\right)^2$ and, by substituting $z = \cos\theta + \mathrm{i}\sin\theta$, find integers p, q, r, s such that

$$64\sin^2\theta\cos^4\theta = p + q\cos 2\theta + r\cos 4\theta + s\cos 6\theta.$$ [6]

Using the substitution $x = 2\cos\theta$, show that

$$\int_1^2 x^4\sqrt{(4-x^2)}\,\mathrm{d}x = \tfrac{4}{3}\pi + \sqrt{3}.$$ [4]

8 The cubic equation $x^3 - x^2 - 3x - 10 = 0$ has roots α, β, γ.

(i) Let $u = -\alpha + \beta + \gamma$. Show that $u + 2\alpha = 1$, and hence find a cubic equation having roots $-\alpha + \beta + \gamma$, $\alpha - \beta + \gamma$, $\alpha + \beta - \gamma$. [5]

(ii) State the value of $\alpha\beta\gamma$ and hence find a cubic equation having roots $\dfrac{1}{\beta\gamma}$, $\dfrac{1}{\gamma\alpha}$, $\dfrac{1}{\alpha\beta}$. [5]

9 The plane Π_1 has parametric equation

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}).$$

Find a cartesian equation of Π_1. [4]

The plane Π_2 has cartesian equation $3x - 2y - 3z = 4$. Find the acute angle between Π_1 and Π_2. [3]

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

10 Find the set of values of a for which the system of equations

$$\begin{aligned} x - 2y - 2z &= -7, \\ 2x + (a-9)y - 10z &= -11, \\ 3x - 6y + 2az &= -29, \end{aligned}$$

has a unique solution. [4]

Show that the system has no solution in the case $a = -3$. [2]

Given that $a = 5$,

(i) show that the number of solutions is infinite, [2]

(ii) find the solution for which $x + y + z = 2$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

The curve C has cartesian equation

$$(x^2 + y^2)^2 = a^2(x^2 - y^2),$$

where a is a positive constant. Show that C has polar equation

$$r^2 = a^2 \cos 2\theta. \qquad [2]$$

Sketch C for $-\pi < \theta \leqslant \pi$. [2]

Find the area of the sector between $\theta = -\frac{1}{4}\pi$ and $\theta = \frac{1}{4}\pi$. [3]

Find the polar coordinates of all points of C where the tangent is parallel to the initial line. [7]

OR

Show that the substitution $y = xz$ reduces the differential equation

$$\frac{1}{x}\frac{d^2y}{dx^2} + \left(\frac{6}{x} - \frac{2}{x^2}\right)\frac{dy}{dx} + \left(\frac{9}{x} - \frac{6}{x^2} + \frac{2}{x^3}\right)y = 169 \sin 2x$$

to the differential equation

$$\frac{d^2z}{dx^2} + 6\frac{dz}{dx} + 9z = 169 \sin 2x. \qquad [4]$$

Find the particular solution for y in terms of x, given that when $x = 0$, $z = -10$ and $\frac{dz}{dx} = 5$. [10]