Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **May/June 2016**

3 hours

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages, **3** blank pages and **1** insert.

CAMBRIDGE International Examinations

1 A bullet of mass 0.01 kg is fired horizontally into a fixed vertical barrier which exerts a constant resisting force of magnitude 1000 N. The bullet enters the barrier with speed $320\,\text{m s}^{-1}$ and emerges with speed $20\,\text{m s}^{-1}$. You may assume that the motion takes place in a horizontal straight line. Find

(i) the magnitude of the impulse that acts on the bullet, [2]

(ii) the thickness of the barrier, [2]

(iii) the time taken for the bullet to pass through the barrier. [1]

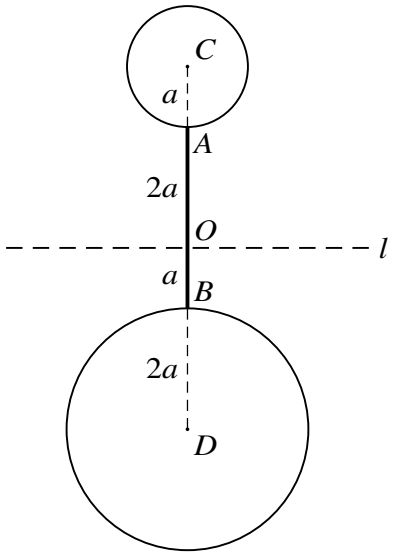
2 A small smooth sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides directly with an identical sphere B which is initially at rest on the surface. The coefficient of restitution between the spheres is e. Sphere B subsequently collides with a fixed vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{1}{2}$. Given that 80% of the initial kinetic energy is lost as a result of the two collisions, find the value of e. [8]

3 A particle P is performing simple harmonic motion with amplitude 0.25 m. During each complete oscillation, P moves with a speed that is less than or equal to half of its maximum speed for $\frac{4}{3}$ seconds. Find the period of the motion and the maximum speed of P. [8]

4 A particle P is at rest at the lowest point on the smooth inner surface of a hollow sphere with centre O and radius a. The particle is projected horizontally with speed u and begins to move in a vertical circle on the inner surface of the sphere. The particle loses contact with the sphere at the point A, where OA makes an angle θ with the upward vertical through O. Given that the speed of P at A is $\sqrt{(\frac{3}{5}ag)}$, find u in terms of a and g. [5]

Find, in terms of a, the greatest height above the level of O achieved by P in its subsequent motion. (You may assume that P achieves its greatest height before it makes any further contact with the sphere.) [5]

5



A thin uniform rod AB has mass $\frac{3}{4}m$ and length $3a$. The end A of the rod is rigidly attached to a point on the circumference of a uniform disc with centre C, mass m and radius a. The end B of the rod is rigidly attached to a point on the circumference of a uniform disc with centre D, mass $4m$ and radius $2a$. The discs and the rod are in the same plane and $CABD$ is a straight line. The mid-point of CD is O. The object consisting of the two discs and the rod is free to rotate about a fixed smooth horizontal axis l, through O in the plane of the object and perpendicular to the rod (see diagram). Show that the moment of inertia of the object about l is $50ma^2$. [6]

The object hangs in equilibrium with D vertically below C. It is displaced through a small angle and released from rest, so that it makes small oscillations about the horizontal axis l. Show that it will move in approximate simple harmonic motion and state the period of the motion. [6]

6 The score when two fair dice are thrown is the sum of the two numbers on the upper faces. Two fair dice are thrown repeatedly until a score of 6 is obtained. The number of throws taken is denoted by the random variable X. Find the mean of X. [2]

Find the least integer N such that the probability of obtaining a score of 6 in fewer than N throws is more than 0.95. [3]

7 A random sample of 9 observations of a normal variable X is taken. The results are summarised as follows.

$$\Sigma x = 24.6 \qquad \Sigma x^2 = 68.5$$

Test, at the 5% significance level, whether the population mean is greater than 2.5. [8]

8 The random variable X has probability density function f given by

$$f(x) = \begin{cases} 2e^{-2x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [2]

(ii) Find the median value of X. [3]

The random variable Y is defined by $Y = e^X$.

(iii) Find the probability density function of Y. [4]

9 Applicants for a national teacher training course are required to pass a mathematics test. Each year, the applicants are tested in groups of 6 and the number of successful applicants in each group is recorded. The overall proportion of successful applicants has remained constant over the years and is equal to 60% of the applicants. The results from 150 randomly chosen groups are shown in the following table.

Number of successful applicants	0	1	2	3	4	5	6
Number of groups	1	3	25	51	38	30	2

Test, at the 5% significance level, the goodness of fit of the distribution B(6, 0.6) for the number of successful applicants in a group. [10]

10 For a random sample of 6 observations of pairs of values (x, y), where $0 < x < 21$ and $0 < y < 14$, the following results are obtained.

$$\Sigma x^2 = 844.20 \qquad \Sigma y^2 = 481.50 \qquad \Sigma xy = 625.59$$

It is also found that the variance of the x-values is 36.66 and the variance of the y-values is 9.69.

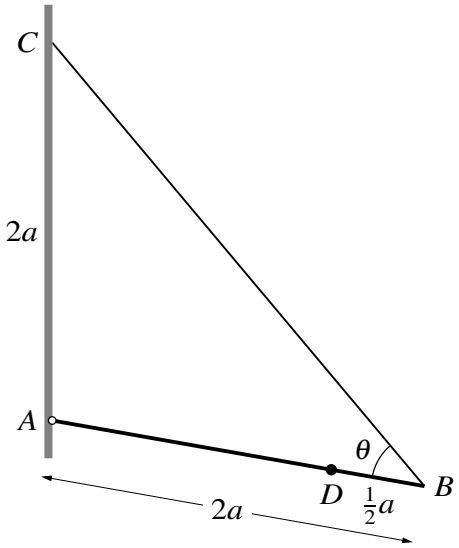
(i) Find the product moment correlation coefficient for the sample. [5]

(ii) Find the equations of the regression lines of y on x and x on y. [4]

(iii) Use the appropriate regression line to estimate the value of x when $y = 6.4$ and comment on the reliability of your estimate. [2]

11 Answer only **one** of the following two alternatives.

EITHER



The end A of a uniform rod AB, of length $2a$ and weight W, is freely hinged to a vertical wall. The end B of the rod is attached to a light elastic string of natural length $\frac{3}{2}a$ and modulus of elasticity $3W$. The other end of the string is attached to the point C on the wall, where C is vertically above A and $AC = 2a$. A particle of weight $2W$ is attached to the rod at the point D, where $DB = \frac{1}{2}a$. The angle ABC is equal to θ (see diagram). Show that $\cos\theta = \frac{3}{4}$ and find the tension in the string in terms of W. [10]

Find the magnitude of the reaction force at the hinge. [4]

OR

Petra is studying a particular species of bird. She takes a random sample of 12 birds from nature reserve A and measures the wing span, x cm, for each bird. She then calculates a 95% confidence interval for the population mean wing span, μ cm, for birds of this species, assuming that wing spans are normally distributed. Later, she is not able to find the summary of the results for the sample, but she knows that the 95% confidence interval is $25.17 \leqslant \mu \leqslant 26.83$. Find the values of Σx and Σx^2 for this sample. [6]

Petra also measures the wing spans of a random sample of 7 birds from nature reserve B. Their wing spans, y cm, are as follows.

23.2 22.4 27.6 25.3 28.4 26.5 23.6

She believes that the mean wing span of birds found in nature reserve A is greater than the mean wing span of birds found in nature reserve B. Assuming that this second sample also comes from a normal distribution, with variance the same as the first distribution, test, at the 10% significance level, whether there is evidence to support Petra's belief. [8]

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