



Cambridge International AS & A Level

CANDIDATE
NAME

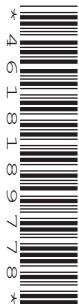
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FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

October/November 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

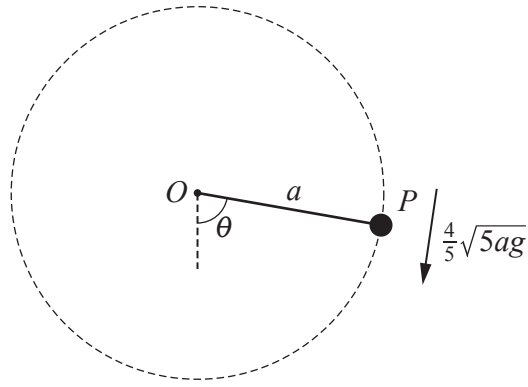
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.



A particle P is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos \theta$. [5]

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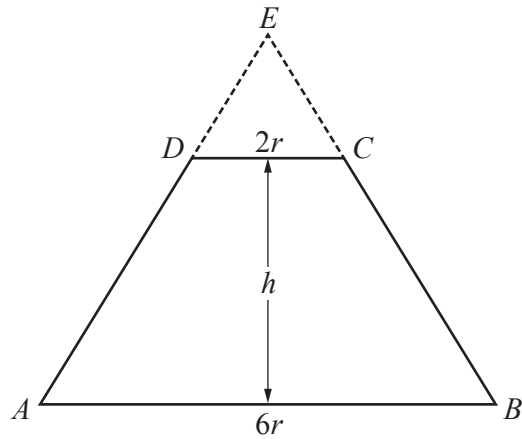
3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O . The length of the string during this motion is $(k + 1)a$.

(a) Find the value of k . [4]

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(b) Find the value of $\cos \theta$. [2]

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The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section DCE from the top of a larger cone with cross-section ABE . The perpendicular distance between AB and DC is h , the diameter AB is $6r$ and the diameter DC is $2r$.

- (a) Find an expression, in terms of h , for the distance of the centre of mass of the solid object from AB . [4]

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5 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha. \quad [3]$$

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The point Q is the highest point on the trajectory of P in the case where $\alpha = 45^\circ$.

(b) Show that the x -coordinate of Q is $\frac{u^2}{2g}$. [3]

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- (c) Find the other value of α for which P would pass through the point Q . [4]

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6 Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B . The coefficient of restitution between the spheres is e .

(a) Find, in terms of u and e , the velocities of A and B after the collision. [3]

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Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B , where $\tan \theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B .

(b) Find the possible values of e . [7]

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7 A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P , in ms^{-2} , is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity 10ms^{-1} directed towards O .

(a) Show that the velocity $v \text{ms}^{-1}$ of P is given by $v = \frac{10(1-2x)}{x}$. [5]

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Additional Page

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