



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

October/November 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 2 A particle P of mass m kg moves along a horizontal straight line with acceleration a ms^{-2} given by

$$a = \frac{v(1-2t^2)}{t},$$

where v ms^{-1} is the velocity of P at time t s.

- (a) Find an expression for v in terms of t and an arbitrary constant. [3]

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- (b) Given that $a = 5$ when $t = 1$, find an expression, in terms of m and t , for the horizontal force acting on P at time t . [3]

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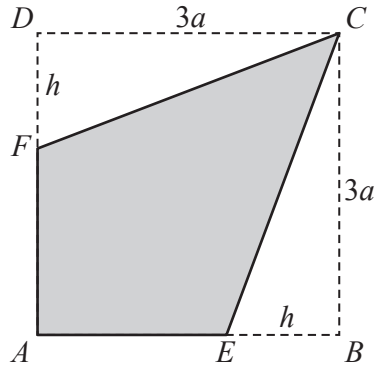
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A uniform lamina $AECF$ is formed by removing two identical triangles BCE and CDF from a square lamina $ABCD$. The square has side $3a$ and $EB = DF = h$ (see diagram).

- (a) Find the distance of the centre of mass of the lamina $AECF$ from AD and from AB , giving your answers in terms of a and h . [5]

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The lamina *AECF* is placed vertically on its edge *AE* on a horizontal plane.

(b) Find, in terms of *a*, the set of values of *h* for which the lamina remains in equilibrium. [3]

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- 6 A particle P , of mass m , is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O . The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is $9 : 5$.

- (a) Find the value of $\cos \theta$. [6]

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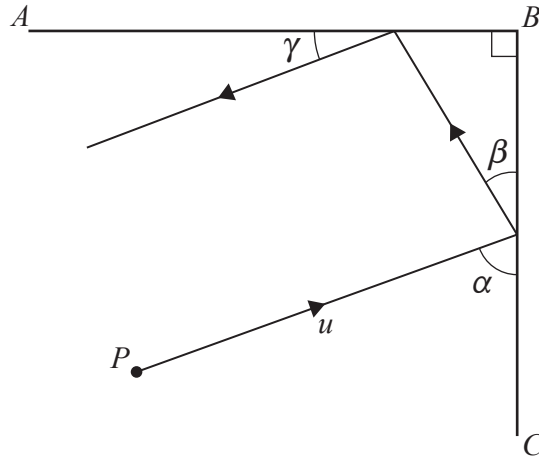
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(b) Find, in terms of a and g , the greatest speed of P during its motion. [2]

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The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α . It rebounds at an angle β to the wall CB . The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e .

- (a) Show that $\tan \beta = e \tan \alpha$. [3]

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- (b) Express γ in terms of α and explain what this result means about the final direction of motion of P . [4]

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As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

- (c) Given that $\alpha + \beta = 90^\circ$, find the value of e and the value of $\tan \alpha$. [4]
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Additional Page

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