Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

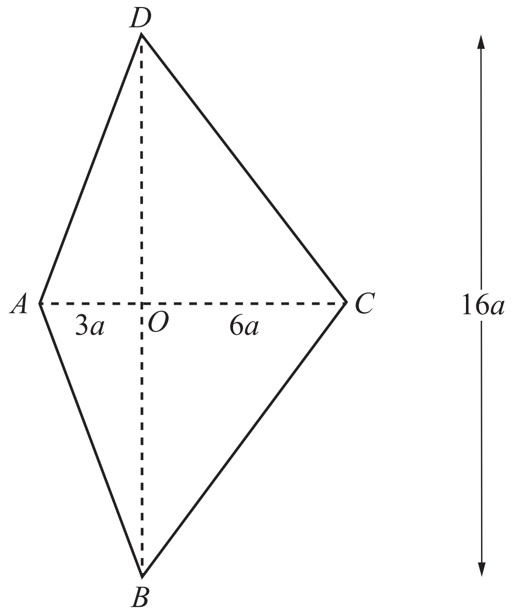
INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (CE/SW) 199008/2

1



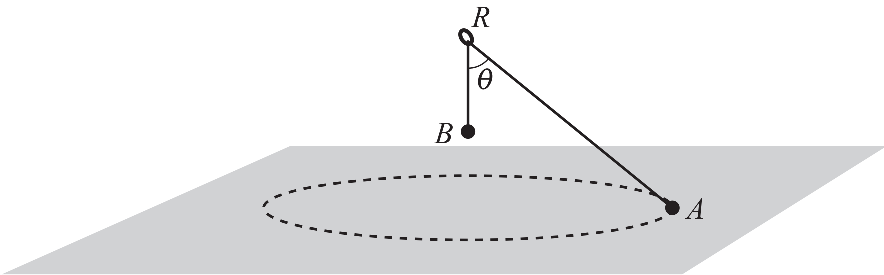
A uniform lamina $ABCD$ consists of two isosceles triangles ABD and BCD. The diagonals of $ABCD$ meet at the point O. The length of AO is $3a$, the length of OC is $6a$ and the length of BD is $16a$ (see diagram).

Find the distance of the centre of mass of the lamina from DB. [3]

2 One end of a light elastic string of natural length $0.8\,\text{m}$ and modulus of elasticity $36\,\text{N}$ is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass $2\,\text{kg}$ is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,\text{m}\,\text{s}^{-1}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]

3



Particles A and B, of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

(a) Find $\cos\theta$. [3]

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(b) Find, in terms of a, the distance of B below the ring. [3]

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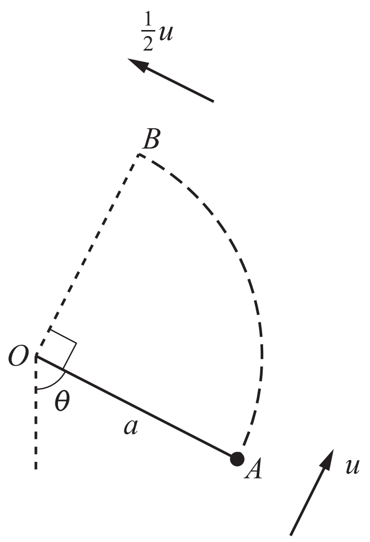

A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is initially held with the string taut at the point A, where OA makes an angle θ with the downward vertical through O. The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point B where angle AOB is a right angle. The speed of the particle when it is at B is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at A, giving your answer in terms of m and g. [8]

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5 A particle P of mass m kg is projected vertically upwards from a point O, with speed $20\,\mathrm{m\,s^{-1}}$, and moves under gravity. There is a resistive force of magnitude $2mv$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of P at time t s after projection.

(a) Find an expression for v in terms of t, while P is moving upwards. [6]

The displacement of P from O is x m at time t s.

(b) Find an expression for x in terms of t, while P is moving upwards. [2]

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(c) Find, correct to 3 significant figures, the greatest height above O reached by P. [2]

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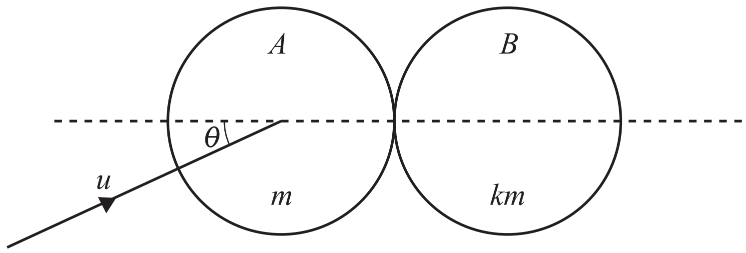

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

(a) Show that the speed of B after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$. [3]

70% of the total kinetic energy of the spheres is lost as a result of the collision.

(b) Given that $\tan\theta = \frac{1}{3}$, find the value of k. [6]

7 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u, θ and g. [2]

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(b) Deduce an expression for the maximum height H, in terms of u, θ and g. [2]

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It is given that $R = \dfrac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60°$. [1]

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It is given also that $u = \sqrt{40}\,\mathrm{m\,s^{-1}}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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Additional Page

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