Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*4190141506*

**FURTHER MATHEMATICS** **9231/11**

Paper 1 Further Pure Mathematics 1 **May/June 2022**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 Let $a$ be a positive constant.

**(a)** Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(ar+1)(ar+a+1)}$ in terms of $n$ and $a$. [4]

**(b)** Find the value of $a$ for which $\sum_{r=1}^{\infty} \frac{1}{(ar+1)(ar+a+1)} = \frac{1}{6}$. [3]

**2** The points $A$, $B$, $C$ have position vectors

$$4\mathbf{i}-4\mathbf{j}+\mathbf{k}, \quad -4\mathbf{i}+3\mathbf{j}-4\mathbf{k}, \quad 4\mathbf{i}-\mathbf{j}-2\mathbf{k},$$

respectively, relative to the origin $O$.

**(a)** Find the equation of the plane $ABC$, giving your answer in the form $ax+by+cz=d$. [5]

**(b)** Find the perpendicular distance from $O$ to the plane $ABC$. [2]

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**(c)** The point $D$ has position vector $2\mathbf{i}+3\mathbf{j}-3\mathbf{k}$.

Find the coordinates of the point of intersection of the line $OD$ with the plane $ABC$. [3]

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**3** The sequence of positive numbers $u_1, u_2, u_3, \ldots$ is such that $u_1 > 4$ and, for $n \geqslant 1$,

$$u_{n+1} = \frac{u_n^2 + u_n + 12}{2u_n}.$$

**(a)** By considering $u_{n+1} - 4$, or otherwise, prove by mathematical induction that $u_n > 4$ for all positive integers $n$. [5]

**(b)** Show that $u_{n+1} < u_n$ for $n \geqslant 1$. [3]

**4** The cubic equation $2x^3 + 5x^2 - 6 = 0$ has roots $\alpha, \beta, \gamma$.

**(a)** Find a cubic equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$. [3]

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**(b)** Find the value of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$. [3]

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**(c)** Find also the value of $\frac{1}{\alpha^9}+\frac{1}{\beta^9}+\frac{1}{\gamma^9}$. [2]

**5** The curve $C$ has equation $y = \dfrac{2x^2 - x - 1}{x^2 + x + 1}$.

**(a)** Show that $C$ has no vertical asymptotes and state the equation of the horizontal asymptote of $C$. [3]

**(b)** Find the coordinates of the stationary points on $C$. [4]

**(c)** Sketch $C$, stating the coordinates of the intersections with the axes. [3]

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**(d)** Sketch the curve with equation $y = \left|\dfrac{2x^2 - x - 1}{x^2 + x + 1}\right|$ and state the set of values of $k$ for which $\left|\dfrac{2x^2 - x - 1}{x^2 + x + 1}\right| = k$ has 4 distinct real solutions. [2]

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**6** The curve $C$ has polar equation $r^2 = \tan^{-1}\left(\frac{1}{2}\theta\right)$, where $0 \leqslant \theta \leqslant 2$.

**(a)** Sketch $C$ and state, in exact form, the greatest distance of a point on $C$ from the pole. [3]

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**(b)** Find the exact value of the area of the region bounded by $C$ and the half-line $\theta = 2$. [5]

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Now consider the part of $C$ where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

**(c)** Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$,

$$(\theta^2 + 4)\tan^{-1}\left(\tfrac{1}{2}\theta\right)\sin\theta - \cos\theta = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

**7** The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

**(a)** Find the set of values of $k$ for which $\mathbf{A}$ is non-singular. [3]

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**(b)** Given that $\mathbf{A}$ is non-singular, find, in terms of $k$, the entries in the top row of $\mathbf{A}^{-1}$. [4]

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**(c)** Given that $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, give an example of a matrix $\mathbf{C}$ such that $\mathbf{BAC} = \begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$. [4]

**(d)** Find the set of values of $k$ for which the transformation in the $x$-$y$ plane represented by $\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ has two distinct invariant lines through the origin. [6]

## Additional page

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