



Cambridge International AS & A Level

CANDIDATE
NAME

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 2 The children at two large schools, P and Q , are all given the same puzzle to solve. A random sample of size 10 is taken from the children at school P . Their individual times to complete the puzzle give a sample mean of 9.12 minutes and an unbiased variance estimate of 2.16 minutes². A random sample of size 12 is taken from the children at school Q . Their individual times, x minutes, to complete the puzzle are summarised by

$$\sum x = 99.6 \qquad \sum (x - \bar{x})^2 = 21.5,$$

where \bar{x} is the sample mean. Times to complete the puzzle are assumed to be normally distributed with the same population variance.

Test at the 5% significance level whether the population mean time taken to complete the puzzle by children at school P is greater than the population mean time taken to complete the puzzle by children at school Q . [8]

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The researcher notices that the figures for company *G* have been recorded incorrectly. In fact, the number of employees in 2018 was 32 and the number of employees in 2022 was 35.

- (b)** Explain, with numerical justification, whether or not the conclusion of the test in part **(a)** remains the same. [2]

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5 Harry has three coins.

- One coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{3}$.
- The second coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{4}$.
- The third coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{5}$.

The random variable X is the number of heads that Harry obtains when he throws all three coins together.

(a) Find the probability generating function of X . [3]

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Isaac has two fair coins. The random variable Y is the number of heads that Isaac obtains when he throws both of his coins together. The random variable Z is the total number of heads obtained when Harry throws his three coins and Isaac throws his two coins.

(b) Find the probability generating function of Z , expressing your answer as a polynomial in t . [4]

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- (c) Use the probability generating function of Z to find $E(Z)$. [2]

6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{3}{28}(e^{\frac{1}{2}x} + 4e^{-\frac{1}{2}x}) & 0 \leq x \leq 2 \ln 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X . [3]

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The random variable Y is defined by $Y = e^{\frac{1}{2}(X)}$.

(b) Find the probability density function of Y . [3]

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(c) Find the 30th percentile of Y .

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(d) Find $E(Y^4)$.

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