Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

*9340929644*

**FURTHER MATHEMATICS** **9231/21**

Paper 2 Further Pure Mathematics 2 **October/November 2023**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.


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**1** Show that the system of equations

$$\begin{aligned} 14x - 4y + 6z &= 5, \\ x + y + kz &= 3, \\ -21x + 6y - 9z &= 14, \end{aligned}$$

where $k$ is a constant, does not have a unique solution and interpret this situation geometrically. [4]

**2** Find the roots of the equation $(z+5\mathrm{i})^3 = 4+4\sqrt{3}\,\mathrm{i}$, giving your answers in the form $r\cos\theta + \mathrm{i}(r\sin\theta - 5)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

**3** Find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\frac{1}{2}e^{x}\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants $a$, $b$ and $c$. [6]

**4** Find the particular solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 27x^2,$$

given that, when $x = 0$, $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -8$. [10]

**5** The curve $C$ has parametric equations

$$x = \tfrac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}, \qquad y = 2t + 5, \qquad \text{for } 0 < t \leqslant 3.$$

**(a)** Find the exact length of $C$. [5]

**(b)** Find the set of values of $t$ for which $\frac{d^2y}{dx^2} > 0$. [5]

**6** **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\sinh 2x = 2\sinh x \cosh x.$$ [3]

**(b)** Using the substitution $u = \sinh x$, find $\int \sinh^2 2x \cosh x \, \mathrm{d}x$. [4]

**(c)** Find the particular solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\tanh x = \sinh^2 2x,$$

given that $y = 4$ when $x = 0$. Give your answer in the form $y = \mathrm{f}(x)$. [7]

**7** The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

**(a)** Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

**(b)** Use the characteristic equation of **A** to find $\mathbf{A}^{-1}$. [4]

**8** **(a)** State the sum of the series $1+z+z^2+\ldots+z^{n-1}$, for $z \neq 1$. [1]

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**(b)** By letting $z = \cos\theta + \mathrm{i}\sin\theta$, where $\cos\theta \neq 1$, show that

$$1+\cos\theta+\cos 2\theta+\ldots+\cos(n-1)\theta = \frac{1}{2}\left(1-\cos n\theta+\frac{\sin n\theta \sin\theta}{1-\cos\theta}\right). \quad [7]$$

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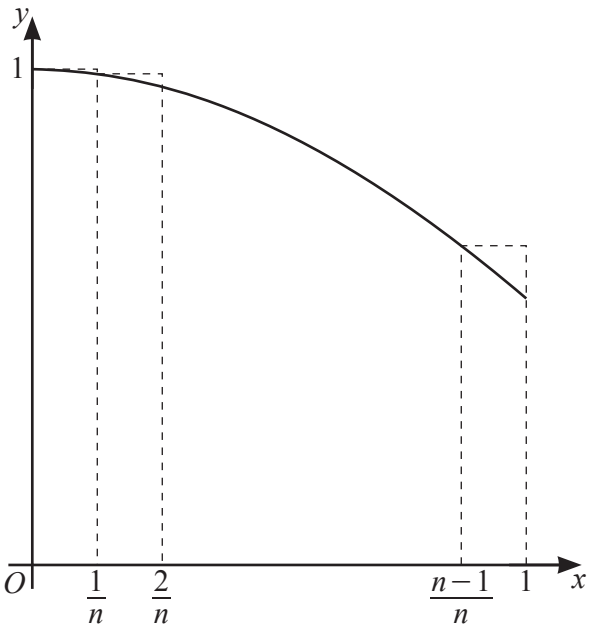


The diagram shows the curve with equation $y = \cos x$ for $0 \leqslant x \leqslant 1$, together with a set of $n$ rectangles of width $\frac{1}{n}$.

**(c)** By considering the sum of the areas of these rectangles, show that

$$\int_0^1 \cos x \mathrm{d}x < \frac{1}{2n}\left(1 - \cos 1 + \frac{\sin 1 \sin \frac{1}{n}}{1 - \cos \frac{1}{n}}\right). \qquad [4]$$

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**(d)** Use a similar method to find, in terms of $n$, a lower bound for $\int_0^1 \cos x \mathrm{d}x$. [3]

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## Additional page

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