

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

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$x = 1 + \frac{1}{t}$	and	$y = te^t$
$\lambda - 1 + \frac{1}{t}$	and	$y - i\epsilon$

(a)	Show that $\frac{dy}{dx} = -e^t(t^3 + t^2)$.	[3]
(b)	Find $\frac{d^2y}{dx^2}$ in terms of t .	[4]

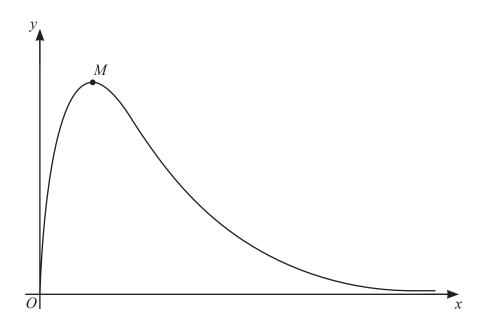
	Use de Moivre's theorem to show that	
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$	[4
)	Hence obtain the roots of the equation	
•)		
	$32x^5 - 40x^3 + 10x - \sqrt{2} = 0$	
	in the form $cos(q\pi)$, where q is a rational number.	[4
	in the form $cos(q\pi)$, where q is a rational number.	[4
	in the form $\cos(q\pi)$, where q is a rational number.	

4 Find the solution of the differential equation

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for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[9]

5



The diagram shows part of the curve $y = x \operatorname{sech}^2 x$ and its maximum point M.

(a) Show that, at M,

$2x \tanh x - 1 = 0$

and verify that this equation has a root between 0.7 and 0.8.	[4]
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$\sum_{n=0}^{\infty} r \sec x$	$ch^2r < n anh$	$n + \ln \operatorname{sech} n$	n – tanh 1 – 1	n sech 1.	
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6	The	matrix	P	is	given	by

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a)	State the eigenvalues of P .	[1]
(b)	Use the characteristic equation of \mathbf{P} to find \mathbf{P}^{-1} .	[4]

The 3×3 matrix **A** has distinct non-zero eigenvalues $a, \frac{1}{2}, 2$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

respectively.

Find A^{-1} in terms of a .	[5]

7	(a)	Starting from the definitions of cosh and sinh in terms of exponentials, prove that	
		$2\sinh^2 A = \cosh 2A - 1.$	[3]
	(b)	A curve has equation $y = x^2$, for $0 \le x \le \frac{2}{3}$. The area of the surface generated when the curve rotated through 2π radians about the x-axis is denoted by S.	/e is
		Use the substitution $x = \frac{1}{2} \sinh u$ to show that $S = \frac{1}{32} \pi \left(\frac{820}{81} - \ln 3 \right)$.	[9]
		32 (61)	
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8 It is given that $v = y^4$ and

$$y^{3} \frac{d^{2}y}{dx^{2}} + 3y^{2} \left(\frac{dy}{dx}\right)^{2} + y^{3} \frac{dy}{dx} + y^{4} = e^{-2x}.$$

(a) Show that

$\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} + \frac{\mathrm{d}v}{\mathrm{d}x} + 4v = 4\mathrm{e}^{-2x}.$	[4]

Find y in terms of x, given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -\frac{3}{8}$.

Additional page

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