Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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5192602583

FURTHER MATHEMATICS **9231/22**

Paper 2 Further Pure Mathematics 2 **October/November 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Find the Maclaurin's series for $\ln(x+2)+\ln(x^2+5)$ up to and including the term in x^2. [5]

2 It is given that

$$x = 1 + \frac{1}{t} \qquad \text{and} \qquad y = te^t.$$

(a) Show that $\frac{dy}{dx} = -e^t(t^3 + t^2)$. [3]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t. [4]

3 **(a)** Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$ [4]

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(b) Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{2} = 0$$

in the form $\cos(q\pi)$, where q is a rational number. [4]

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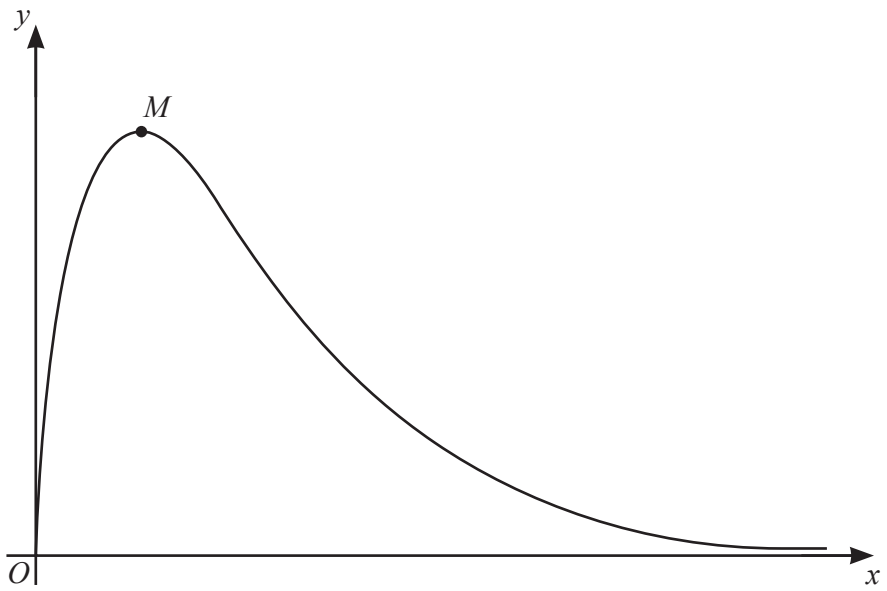
4 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \sin x$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = \mathrm{f}(x)$. [9]

5



The diagram shows part of the curve $y = x\,\text{sech}^2 x$ and its maximum point M.

(a) Show that, at M,

$$2x\tanh x - 1 = 0$$

and verify that this equation has a root between 0.7 and 0.8. [4]

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(b) By considering a suitable set of rectangles, use the diagram to show that

$$\sum_{r=2}^{n} r\,\mathrm{sech}^2 r < n\tanh n + \ln\mathrm{sech}\, n - \tanh 1 - \ln\mathrm{sech}\, 1. \qquad [6]$$

6 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

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(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

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The 3×3 matrix $\mathbf{A}$ has distinct non-zero eigenvalues a, $\frac{1}{2}$, 2 with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

respectively.

(c) Find $\mathbf{A}^{-1}$ in terms of a. [5]

7 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$2\sinh^2 A = \cosh 2A - 1. \qquad [3]$$

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(b) A curve has equation $y = x^2$, for $0 \leqslant x \leqslant \frac{2}{3}$. The area of the surface generated when the curve is rotated through 2π radians about the x-axis is denoted by S.

Use the substitution $x = \frac{1}{2}\sinh u$ to show that $S = \frac{1}{32}\pi\left(\frac{820}{81} - \ln 3\right)$. [9]

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[Turn over

8 It is given that $v = y^4$ and

$$y^3 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3y^2 \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + y^3 \frac{\mathrm{d}y}{\mathrm{d}x} + y^4 = \mathrm{e}^{-2x}.$$

(a) Show that

$$\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} + \frac{\mathrm{d}v}{\mathrm{d}x} + 4v = 4\mathrm{e}^{-2x}. \qquad [4]$$

(b) Find y in terms of x, given that, when $x = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{8}$. [10]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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