Cambridge Assessment International Education

# Cambridge International AS & A Level

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*0815246258*

**FURTHER MATHEMATICS** **9231/42**

Paper 4 Further Probability & Statistics **October/November 2023**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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**1** A factory produces small bottles of natural spring water. Two different machines, $X$ and $Y$, are used to fill empty bottles with the water. A quality control engineer checks the volumes of water in the bottles filled by each of the machines. He chooses a random sample of 60 bottles filled by machine $X$ and a random sample of 75 bottles filled by machine $Y$. The volumes of water, $x$ and $y$ respectively, in millilitres, are summarised as follows.

$$\Sigma x = 6345 \quad \Sigma(x-\overline{x})^2 = 243.8 \quad \Sigma y = 7614 \quad \Sigma(y-\overline{y})^2 = 384.9$$

$\overline{x}$ and $\overline{y}$ are the sample means of the volume of water in the bottles filled by machines $X$ and $Y$ respectively.

Find a 95% confidence interval for the difference between the mean volume of water in bottles filled by machine $X$ and the mean volume of water in bottles filled by machine $Y$. [6]

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**[Turn over**

**2** The number of breakdowns on a particular section of road is recorded each day over a period of 90 days. It is suggested that the number of breakdowns follows a Poisson distribution with mean 3.5. The data is summarised in the table, together with some of the expected frequencies resulting from the suggested Poisson distribution.

| Number of breakdowns per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 or more |
|---|---|---|---|---|---|---|---|---|---|
| Observed frequency | 0 | 5 | 13 | 17 | 21 | 16 | 9 | 5 | 4 |
| Expected frequency | 2.718 | 9.512 | 16.646 | | 16.993 | 11.895 | | 3.469 | 2.407 |

**(a)** Complete the table. [2]

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**(b)** Carry out a goodness of fit test, at the 10% significance level, to determine whether or not Po(3.5) is a good fit to the data. [6]

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**3** Toby has a bag which contains 6 red marbles and 3 green marbles. He randomly chooses 3 marbles from the bag, without replacement. The random variable $X$ is the number of red marbles that Toby obtains.

**(a)** Find the probability generating function of $X$. [3]

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Ling also has a bag which contains 6 red marbles and 3 green marbles. He randomly chooses 2 marbles from his bag, without replacement. The random variable $Y$ is the number of red marbles that Ling obtains. It is given that the probability generating function of $Y$ is $\frac{1}{12}(1+6t+5t^2)$.

The random variable $Z$ is the total number of red marbles that Toby and Ling obtain.

**(b)** Find the probability generating function of $Z$, expressing your answer as a polynomial in $t$. [3]

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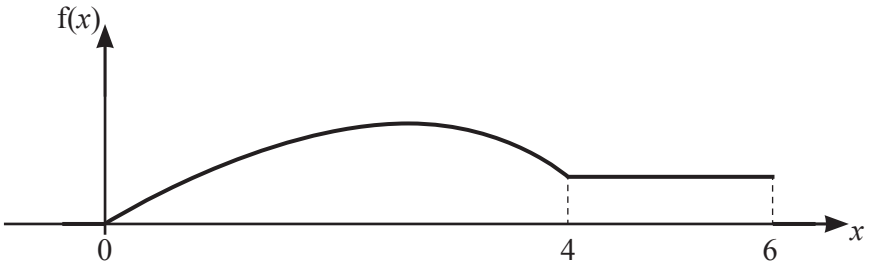
**(c)** Use the probability generating function of $Z$ to find $\text{Var}(Z)$. [4]

**4**




The diagram shows the continuous random variable $X$ with probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{128}(4ax - bx^3) & 0 \leqslant x \leqslant 4, \\ c & 4 \leqslant x \leqslant 6, \\ 0 & \text{otherwise}, \end{cases}$$

where $a$, $b$ and $c$ are constants.

The upper quartile of $X$ is equal to 4.

**(a)** Show that $c = \frac{1}{8}$ and find the values of $a$ and $b$. [4]

**(b)** Find the exact value of the median of $X$. [3]

**(c)** Find $\mathrm{E}(\sqrt{X})$, giving your answer correct to 2 decimal places. [3]

**5** A company is deciding which of two machines, $X$ and $Y$, can make a certain type of electrical component more quickly. The times taken, in minutes, to make one component of this type are recorded for a random sample of 8 components made by machine $X$ and a random sample of 9 components made by machine $Y$. These times are as follows.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Machine $X$ | 4.0 | 4.6 | 4.7 | 4.8 | 5.0 | 5.2 | 5.6 | 5.8 | |
| Machine $Y$ | 4.5 | 4.9 | 5.1 | 5.3 | 5.4 | 5.7 | 5.9 | 6.3 | 6.4 |

The manager claims that on average the time taken by machine $X$ to make one component is less than that taken by machine $Y$.

**(a)** Carry out a Wilcoxon rank-sum test at the 5% significance level to test whether the manager's claim is supported by the data. [6]

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**(b)** Assuming that the times taken to produce the components by the two machines are normally distributed with equal variances, carry out a *t*-test at the 5% significance level to test whether the manager's claim is supported by the data. [9]

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**Question 5(c) is printed on the next page.**

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**(c)** In general, would you expect the conclusions from the tests in parts **(a)** and **(b)** to be the same? Give a reason for your answer. [1]

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## Additional page

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