



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

3 A factory produces metal discs. The manager claims that the diameters of these discs have a median of 22.0 mm. The diameters, in mm, of a random sample of 12 discs produced by this factory are as follows.

22.4 20.9 22.8 21.5 23.2 22.9 23.9 21.7 19.8 23.6 22.6 23.0

(a) Carry out a Wilcoxon signed-rank test, at the 10% significance level, to test whether there is any evidence against the manager’s claim. [7]

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(b) State an assumption that is necessary for this test to be valid. [1]

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- 4 The random variable Y is the sum of two independent observations of the random variable X . The probability generating function $G_Y(t)$ of Y is given by

$$G_Y(t) = \frac{t^2}{(4-3t)^4}.$$

- (a) Find $E(Y)$. [3]

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- (b) Write down an expression for the probability generating function of X . [1]

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(c) Find $P(X = 4)$.

[3]

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5 Two companies, P and Q , produce a certain type of paint brush. An independent examiner rates the quality of the brushes produced as poor, satisfactory or good. He takes a random sample of brushes from each company. The examiner’s ratings are summarised in the table.

Company	Poor	Satisfactory	Good
P	18	43	64
Q	22	22	31

(a) Test, at the 5% significance level, whether quality of brushes is independent of company. [7]

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- 6 Jade is a swimming instructor at a sports college. She claims that, as a result of an intensive training course, the mean time taken by students to swim 50 metres has reduced by more than 1 second. She chooses a random sample of 10 students. The times taken, in seconds, before and after the training course are recorded in the table.

Student	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>
Time before course	54.2	47.4	52.1	59.0	55.3	51.0	48.9	52.2	58.4	51.4
Time after course	50.1	46.3	52.5	58.8	51.4	48.4	49.5	48.7	58.3	51.4

- (a) Test, at the 10% significance level, whether Jade’s claim is justified. [7]

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7 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{x}{4}(4-x^2) & 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\text{Var}(\sqrt{X})$.

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The continuous random variable Y is defined by $Y = X^2$.

- (b) Find the probability density function of Y . [4]

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- (c) Find the exact value of the median of Y . [2]

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Additional page

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