Differential Equations

S11(11)

7 The variables x and y are related by the differential equation

$$y^2\frac{d^2y}{dx^2} + 2y^2\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right)^2 - 5y^3 = 8e^{-x}.$$

Given that $v = y^3$, show that

$$\frac{d^2v}{dx^2} + 2\frac{dv}{dx} - 15v = 24e^{-x}.$$ [4]

Hence find the general solution for y in terms of x. [7]

S11(13)

8 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10\sin t.$$ [6]

Find the particular solution, given that $x = 5$ and $\frac{dx}{dt} = 2$ when $t = 0$. [4]

State an approximate solution for large positive values of t. [1]

S12(11)

8 Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10e^{-2x},$$

given that $y = 5$ and $\frac{dy}{dx} = 1$ when $x = 0$. [11]

S12(13) Question 11

OR

Show that the substitution $y = xz$ reduces the differential equation

$$\frac{1}{x}\frac{d^2y}{dx^2} + \left(\frac{6}{x} - \frac{2}{x^2}\right)\frac{dy}{dx} + \left(\frac{9}{x} - \frac{6}{x^2} + \frac{2}{x^3}\right)y = 169\sin 2x$$

to the differential equation

$$\frac{d^2z}{dx^2} + 6\frac{dz}{dx} + 9z = 169\sin 2x.$$ [4]

Find the particular solution for y in terms of x, given that when $x = 0$, $z = -10$ and $\frac{dz}{dx} = 5$. [10]

S13(11)

9 Find x in terms of t given that

$$4\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x = 6e^{-2t},$$

and that, when $t = 0$, $x = \frac{5}{3}$ and $\frac{dx}{dt} = \frac{7}{6}$. [9]

State $\lim_{t\to\infty} x$. [1]

S13(13)

7 Find the value of the constant λ such that $\lambda x e^{-x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 6e^{-x}.$$ [4]

Find the solution of the differential equation for which $y = 2$ and $\frac{dy}{dx} = 3$ when $x = 0$. [6]

W11(11) Question 11

OR

Given that

$$x^2\frac{d^2y}{dx^2} + 4x(1 + x)\frac{dy}{dx} + 2(1 + 4x + 2x^2)y = 8x^2$$

and that $x^2y = z$, show that

$$\frac{d^2z}{dx^2} + 4\frac{dz}{dx} + 4z = 8x^2.$$ [4]

Find the general solution for y in terms of x. [8]

Describe the behaviour of y as $x \to \infty$. [2]

W11(13)

6 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = \sin 2t.$$ [6]

Describe the behaviour of x as $t \to \infty$, justifying your answer. [2]

W12(11)

3 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 26t^2 + 3t + 13.$$ [6]

W12(13) Question 11

OR

Obtain the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 75\cos 2t.$$ [7]

Given that $x = 5$ and $\frac{dx}{dt} = 0$ when $t = 0$, find x in terms of t. [4]

Show that, for large positive values of t and for any initial conditions,

$$x \approx 5\cos(2t - \phi),$$

where the constant ϕ is such that $\tan\phi = \frac{4}{3}$. [3]

W13(11)

OR

Given that

$$y^2\frac{d^2y}{dx^2} - 6y^2\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right)^2 + 3y^3 = 25e^{-2x}$$

and that $v = y^3$, show that

$$\frac{d^2v}{dx^2} - 6\frac{dv}{dx} + 9v = 75e^{-2x}.$$ [4]

Find the particular solution for y in terms of x, given that when $x = 0$, $y = 2$ and $\frac{dy}{dx} = 1$. [10]

W13(13)

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 4x^2 + 8.$$ [7]