Past Year: Chapter 5 Trigonometry

May/June 2002

2 **(i)** Show that $\sin x \tan x$ may be written as $\dfrac{1-\cos^2 x}{\cos x}$. [1]

(ii) Hence solve the equation $2\sin x \tan x = 3$, for $0° \leqslant x \leqslant 360°$. [4]

6 The function f, where $f(x) = a\sin x + b$, is defined for the domain $0 \leqslant x \leqslant 2\pi$. Given that $f(\frac{1}{2}\pi) = 2$ and that $f(\frac{3}{2}\pi) = -8$,

(i) find the values of a and b, [3]

(ii) find the values of x for which $f(x) = 0$, giving your answers in radians correct to 2 decimal places, [2]

(iii) sketch the graph of $y = f(x)$. [2]

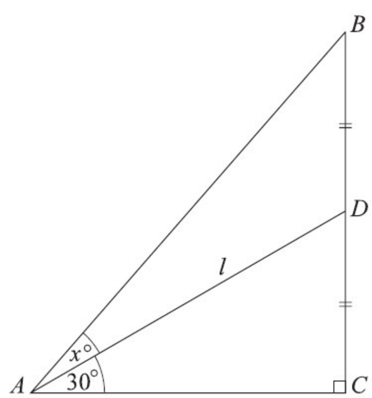
Nov/Dec 2002

5 **(i)** Show that the equation $3\tan\theta = 2\cos\theta$ can be expressed as

$$2\sin^2\theta + 3\sin\theta - 2 = 0.$$ [3]

(ii) Hence solve the equation $3\tan\theta = 2\cos\theta$, for $0° \leqslant \theta \leqslant 360°$. [3]

6



In the diagram, triangle ABC is right-angled and D is the mid-point of BC. Angle $DAC = 30°$ and angle $BAD = x°$. Denoting the length of AD by l,

(i) express each of AC and BC exactly in terms of l, and show that $AB = \frac{1}{2}l\sqrt{7}$, [4]

(ii) show that $x = \tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right) - 30$. [2]

May/June 03

2 Find all the values of x in the interval $0° \leqslant x \leqslant 180°$ which satisfy the equation $\sin 3x + 2\cos 3x = 0$. [4]

May/June 2004

3 **(i)** Show that the equation $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ can be written as a quadratic equation in $\tan\theta$. [2]

(ii) Hence, or otherwise, solve the equation in part **(i)** for $0° \leqslant \theta \leqslant 180°$. [3]

May/June 2005

3 (i) Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. [2]

(ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0° \leqslant \theta \leqslant 360°$. [2]

7 A function f is defined by $f : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant 360°$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by $g : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse. [1]

(iv) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$. [2]

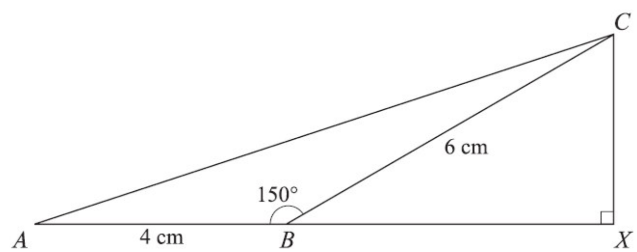
May/June 2006

2 Solve the equation

$$\sin 2x + 3\cos 2x = 0,$$

for $0° \leqslant x \leqslant 180°$. [4]

6



In the diagram, ABC is a triangle in which $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150°$. The line CX is perpendicular to the line ABX.

(i) Find the exact length of BX and show that angle $CAB = \tan^{-1}\left(\frac{3}{4 + 3\sqrt{3}}\right)$. [4]

(ii) Show that the exact length of AC is $\sqrt{(52 + 24\sqrt{3})}$ cm. [2]

May/June 2007

3 Prove the identity $\frac{1 - \tan^2 x}{1 + \tan^2 x} \equiv 1 - 2\sin^2 x$. [4]

8 The function f is defined by $f(x) = a + b\cos 2x$, for $0 \leqslant x \leqslant \pi$. It is given that $f(0) = -1$ and $f(\frac{1}{2}\pi) = 7$.

(i) Find the values of a and b. [3]

(ii) Find the x-coordinates of the points where the curve $y = f(x)$ intersects the x-axis. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

May/June 2008

1 In the triangle ABC, $AB = 12$ cm, angle $BAC = 60°$ and angle $ACB = 45°$. Find the exact length of BC. [3]

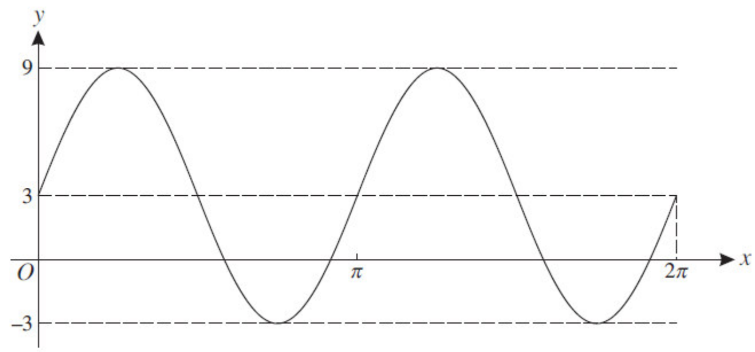
2 **(i)** Show that the equation $2\tan^2\theta\cos\theta = 3$ can be written in the form $2\cos^2\theta + 3\cos\theta - 2 = 0$. [2]

(ii) Hence solve the equation $2\tan^2\theta\cos\theta = 3$, for $0° \leqslant \theta \leqslant 360°$. [3]

May/June 2009

1 Prove the identity $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x} \equiv 2\tan^2 x$. [3]

4



The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$.

(i) Find the values of a, b and c. [3]

(ii) Find the smallest value of x in the interval $0 \leqslant x \leqslant 2\pi$ for which $y = 0$. [3]

Oct/Nov 2001

3 **(i)** Sketch and label, on the same diagram, the graphs of $y = \cos x$ and $y = \cos 3x$ for the interval $0 \leqslant x \leqslant 2\pi$. [3]

(ii) Given that $f : x \mapsto \cos x$, for the domain $0 \leqslant x \leqslant k$, find the largest value of k for which f has an inverse. [2]

7 It is given that $a = 2\sin\theta + \cos\theta$ and $b = 2\cos\theta - \sin\theta$, where $0° \leqslant \theta \leqslant 360°$.

(i) Show that $a^2 + b^2$ is constant for all values of θ. [3]

(ii) Given that $2a = 3b$, show that $\tan\theta = \frac{4}{7}$ and find the corresponding values of θ. [4]

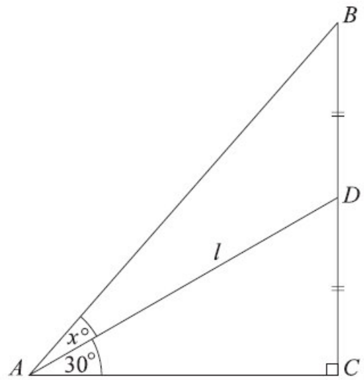
Oct/Nov 2002

5 **(i)** Show that the equation $3\tan\theta = 2\cos\theta$ can be expressed as

$$2\sin^2\theta + 3\sin\theta - 2 = 0.$$ [3]

(ii) Hence solve the equation $3\tan\theta = 2\cos\theta$, for $0° \leqslant \theta \leqslant 360°$. [3]

6



In the diagram, triangle ABC is right-angled and D is the mid-point of BC. Angle $DAC = 30°$ and angle $BAD = x°$. Denoting the length of AD by l,

(i) express each of AC and BC exactly in terms of l, and show that $AB = \frac{1}{2}l\sqrt{7}$, [4]

(ii) show that $x = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - 30$. [2]

Oct/Nov 2003

2 **(i)** Show that the equation $4\sin^4\theta + 5 = 7\cos^2\theta$ may be written in the form $4x^2 + 7x - 2 = 0$, where $x = \sin^2\theta$. [1]

(ii) Hence solve the equation $4\sin^4\theta + 5 = 7\cos^2\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

Oct/Nov 2004

6 The function $f : x \mapsto 5\sin^2 x + 3\cos^2 x$ is defined for the domain $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\sin^2 x$, stating the values of a and b. [2]

(ii) Hence find the values of x for which $f(x) = 7\sin x$. [3]

(iii) State the range of f. [2]

Oct/Nov 2005

1 Solve the equation $3\sin^2\theta - 2\cos\theta - 3 = 0$, for $0° \leqslant \theta \leqslant 180°$. [4]

Oct/Nov 2006

2 Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$, [2]

(ii) $\tan^2 x$. [2]

Oct/Nov 2007

5 **(i)** Show that the equation $3\sin x\tan x = 8$ can be written as $3\cos^2 x + 8\cos x - 3 = 0$. [3]

(ii) Hence solve the equation $3\sin x\tan x = 8$ for $0° \leqslant x \leqslant 360°$. [3]

Oct/Nov 2008

2 Prove the identity

$$\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \equiv \frac{2}{\cos x}.$$ [4]

5 The function f is such that $f(x) = a - b\cos x$ for $0° \leqslant x \leqslant 360°$, where a and b are positive constants. The maximum value of $f(x)$ is 10 and the minimum value is -2.

(i) Find the values of a and b. [3]

(ii) Solve the equation $f(x) = 0$. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

Oct/Nov 2009/11

1 Solve the equation $3\tan(2x + 15°) = 4$ for $0° \leqslant x \leqslant 180°$. [4]

2 The equation of a curve is $y = 3\cos 2x$. The equation of a line is $x + 2y = \pi$. On the same diagram, sketch the curve and the line for $0 \leqslant x \leqslant \pi$. [4]

Oct/Nov 2009/12

4 The function f is defined by $f : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State, with a reason, whether f has an inverse. [1]

5 **(i)** Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. [3]

(ii) Solve the equation $(\sin x + \cos x)(1 - \sin x \cos x) = 9\sin^3 x$ for $0° \leqslant x \leqslant 360°$. [3]

May/June 2010/11

1 The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$, [1]

(ii) $\tan(\frac{1}{2}\pi - x)$, [1]

(iii) $\sin x$. [2]

5 The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b. [2]

(ii) State the greatest and least values of $f(x)$. [2]

(iii) Solve the equation $f(x) + 1 = 0$. [3]

May/June 2010/12

1 **(i)** Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0° \leqslant x \leqslant 360°$. [2]

11 The function $f : x \mapsto 4 - 3\sin x$ is defined for the domain $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 2$. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [2]

The function $g : x \mapsto 4 - 3\sin x$ is defined for the domain $\frac{1}{2}\pi \leqslant x \leqslant A$.

(iv) State the largest value of A for which g has an inverse. [1]

(v) For this value of A, find the value of $g^{-1}(3)$. [2]

May/June 2010/13

3 The function $f : x \mapsto a + b\cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that $f(0) = 10$ and that $f\left(\frac{2}{3}\pi\right) = 1$, find

(i) the values of a and b, [2]

(ii) the range of f, [1]

(iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]

4 **(i)** Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2\cos^2 x - 3\cos x - 2 = 0$. [2]

(ii) Solve the equation $2\sin x \tan x + 3 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

Oct/Nov 2010/11

4 **(i)** Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\dfrac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0° \leqslant x \leqslant 360°$. [3]

7 A function f is defined by $f : x \mapsto 3 - 2\tan\left(\frac{1}{2}x\right)$ for $0 \leqslant x < \pi$.

(i) State the range of f. [1]

(ii) State the exact value of $f\left(\frac{2}{3}\pi\right)$. [1]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Obtain an expression, in terms of x, for $f^{-1}(x)$. [3]

Oct/Nov 2010/12

2 Prove the identity

$$\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x.$$ [4]

Oct/Nov 2010/13

3 Solve the equation $15\sin^2 x = 13 + \cos x$ for $0° \leqslant x \leqslant 180°$. [4]

4 **(i)** Sketch the curve $y = 2\sin x$ for $0 \leqslant x \leqslant 2\pi$. [1]

(ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation

$$2\pi \sin x = \pi - x.$$

State the equation of the straight line. [3]