Past Year: Chapter 9 Integration

May/June 2002

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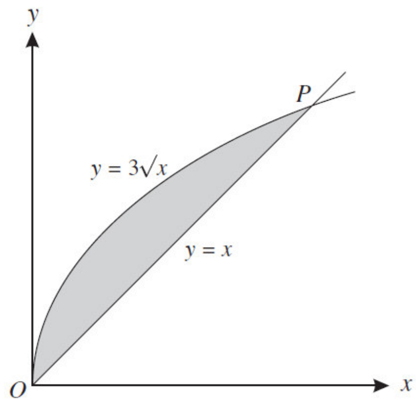

The diagram shows the curve $y = 3\sqrt{x}$ and the line $y = x$ intersecting at O and P. Find

(i) the coordinates of P, [1]

(ii) the area of the shaded region. [5]

9 A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$ and $P(1, 5)$ is a point on the curve.

(i) The normal to the curve at P crosses the x-axis at Q. Find the coordinates of Q. [4]

(ii) Find the equation of the curve. [4]

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of increase of the y-coordinate when $x = 1$. [3]

Nov/Dec 2002

4 The gradient at any point (x, y) on a curve is $\sqrt{(1 + 2x)}$. The curve passes through the point $(4, 11)$. Find

(i) the equation of the curve, [4]

(ii) the point at which the curve intersects the y-axis. [2]

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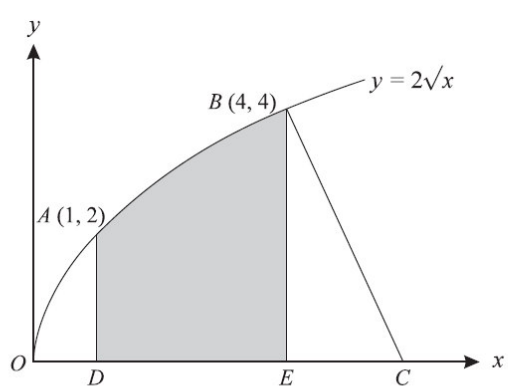


The diagram shows the points $A(1, 2)$ and $B(4, 4)$ on the curve $y = 2\sqrt{x}$. The line BC is the normal to the curve at B, and C lies on the x-axis. Lines AD and BE are perpendicular to the x-axis.

(i) Find the equation of the normal BC. [4]

(ii) Find the area of the shaded region. [4]

May/June 03

3 **(a)** Differentiate $4x + \frac{6}{x^2}$ with respect to x. [2]

(b) Find $\int \left(4x + \frac{6}{x^2}\right) \mathrm{d}x$. [3]

10 The equation of a curve is $y = \sqrt{(5x + 4)}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

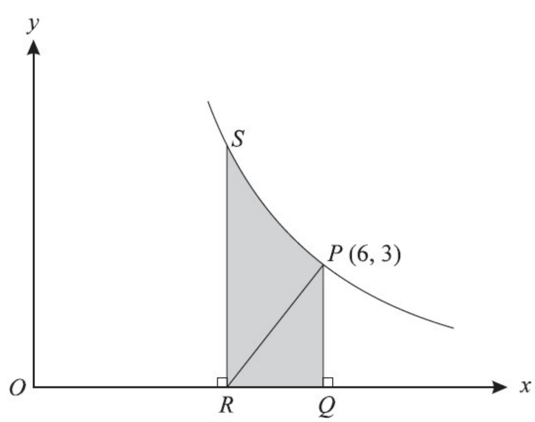
(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of x has the constant value 0.03 units per second. Find the rate of increase of y at the instant when $x = 1$. [2]

(iii) Find the area enclosed by the curve, the x-axis, the y-axis and the line $x = 1$. [5]

May/June 2004

2 Evaluate $\int_0^1 \sqrt{(3x + 1)}\, \mathrm{d}x$. [4]

7



The diagram shows part of the graph of $y = \frac{18}{x}$ and the normal to the curve at $P\,(6, 3)$. This normal meets the x-axis at R. The point Q on the x-axis and the point S on the curve are such that PQ and SR are parallel to the y-axis.

(i) Find the equation of the normal at P and show that R is the point $(4\frac{1}{2}, 0)$. [5]

(ii) Show that the volume of the solid obtained when the shaded region $PQRS$ is rotated through $360°$ about the x-axis is 18π. [4]

May/June 2005

1 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^2 - 5$. Given that the point $(3, 8)$ lies on the curve, find the equation of the curve. [4]

9 A curve has equation $y = \frac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point $(4, 2)$ meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$. [4]

May/June 2006

9 A curve is such that $\frac{dy}{dx} = \frac{4}{\sqrt{(6-2x)}}$, and $P(1, 8)$ is a point on the curve.

(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR. [5]

(ii) Find the equation of the curve. [4]

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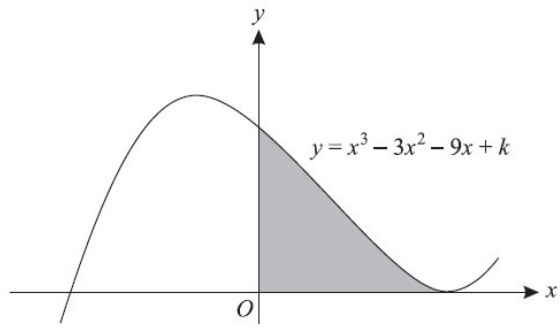


The diagram shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.

(i) Find the value of k. [4]

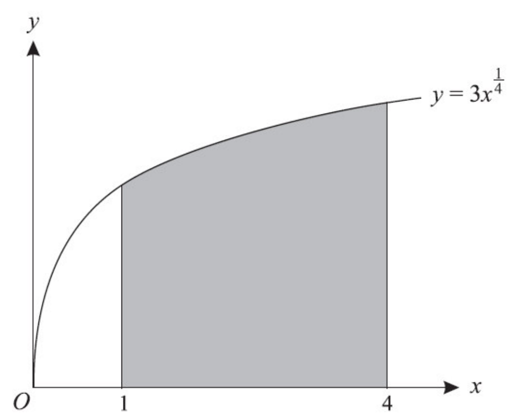
(ii) Find the coordinates of the maximum point of the curve. [1]

(iii) State the set of values of x for which $x^3 - 3x^2 - 9x + k$ is a decreasing function of x. [1]

(iv) Find the area of the shaded region. [4]

May/June 2007

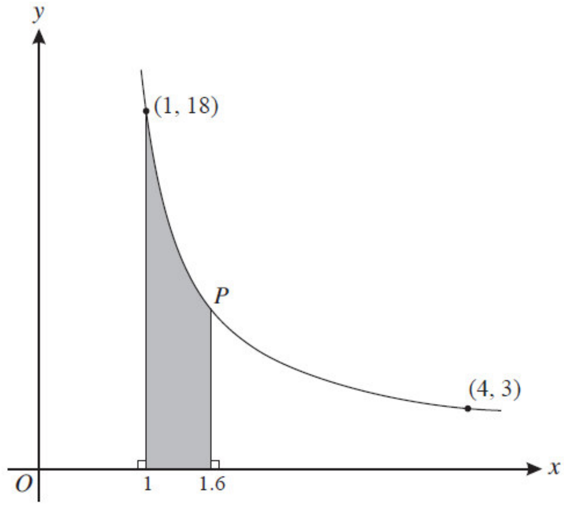
2



The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π. [4]

May/June 2008

9



The diagram shows a curve for which $\frac{dy}{dx} = -\frac{k}{x^3}$, where k is a constant. The curve passes through the points (1, 18) and (4, 3).

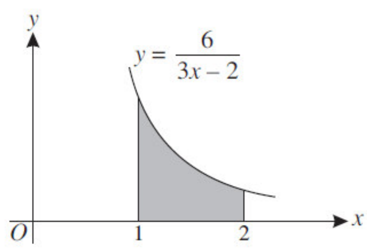
(i) Show, by integration, that the equation of the curve is $y = \frac{16}{x^2} + 2$. [4]

The point P lies on the curve and has x-coordinate 1.6.

(ii) Find the area of the shaded region. [4]

May/June 2009

9

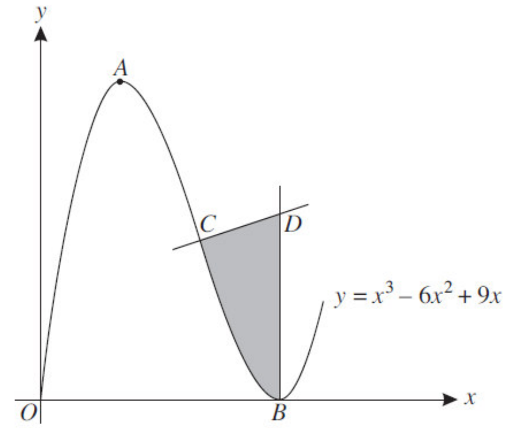


The diagram shows part of the curve $y = \frac{6}{3x-2}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [5]

11



The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geqslant 0$. The curve has a maximum point at A and a minimum point on the x-axis at B. The normal to the curve at $C\ (2, 2)$ meets the normal to the curve at B at the point D.

(i) Find the coordinates of A and B. [3]

(ii) Find the equation of the normal to the curve at C. [3]

(iii) Find the area of the shaded region. [5]

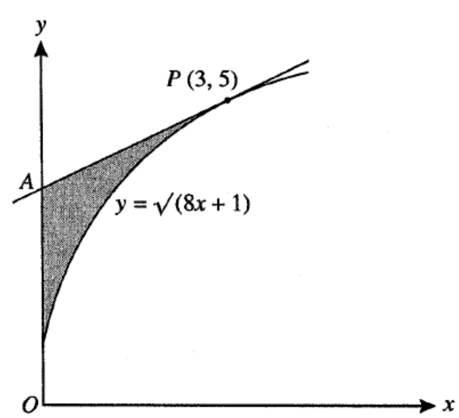
Oct/Nov 2001

9 A curve is such that $\frac{dy}{dx} = \frac{24}{x^3} - 3$.

(i) Given that the curve passes through the point $(1, 16)$, find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [4]

11



The diagram shows the curve $y = \sqrt{(8x + 1)}$ and the tangent at the point $P\,(3, 5)$ on the curve. This tangent meets the y-axis at A. Find

(i) the equation of the tangent at P, [4]

(ii) the coordinates of A, [1]

(iii) the area of the shaded region. [6]

Oct/Nov 2002

4 The gradient at any point (x, y) on a curve is $\sqrt{(1 + 2x)}$. The curve passes through the point $(4, 11)$. Find

(i) the equation of the curve, [4]

(ii) the point at which the curve intersects the y-axis. [2]

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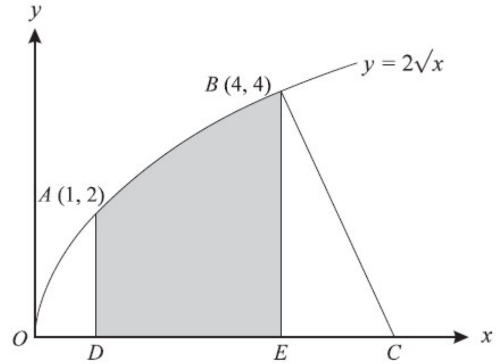


The diagram shows the points A (1, 2) and B (4, 4) on the curve $y = 2\sqrt{x}$. The line BC is the normal to the curve at B, and C lies on the x-axis. Lines AD and BE are perpendicular to the x-axis.

(i) Find the equation of the normal BC. [4]

(ii) Find the area of the shaded region. [4]

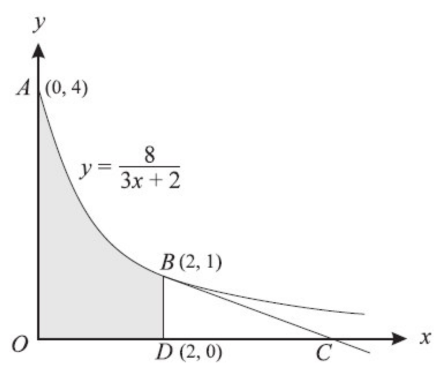
Oct/Nov 2003

4 A curve is such that $\frac{dy}{dx} = 3x^2 - 4x + 1$. The curve passes through the point (1, 5).

(i) Find the equation of the curve. [3]

(ii) Find the set of values of x for which the gradient of the curve is positive. [3]

9



The diagram shows points A (0, 4) and B (2, 1) on the curve $y = \frac{8}{3x+2}$. The tangent to the curve at B crosses the x-axis at C. The point D has coordinates (2, 0).

(i) Find the equation of the tangent to the curve at B and hence show that the area of triangle BDC is $\frac{4}{3}$. [6]

(ii) Show that the volume of the solid formed when the shaded region $ODBA$ is rotated completely about the x-axis is 8π. [5]

Oct/Nov 2004

7 A curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(4x-3)}}$ and P (3, 3) is a point on the curve.

(i) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$. [3]

(ii) Find the equation of the curve. [4]

10 A curve has equation $y = x^2 + \frac{2}{x}$.

(i) Write down expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(iii) Find the volume of the solid formed when the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated completely about the x-axis. [6]

Oct/Nov 2005

10 A curve is such that $\frac{dy}{dx} = \frac{16}{x^3}$, and (1, 4) is a point on the curve.

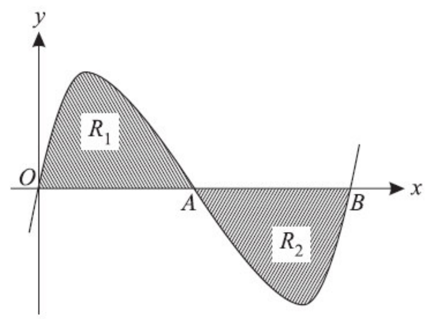
(i) Find the equation of the curve. [4]

(ii) A line with gradient $-\frac{1}{2}$ is a normal to the curve. Find the equation of this normal, giving your answer in the form $ax + by = c$. [4]

(iii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [4]

Oct/Nov 2006

7



The diagram shows the curve $y = x(x - 1)(x - 2)$, which crosses the x-axis at the points O (0, 0), A (1, 0) and B (2, 0).

(i) The tangents to the curve at the points A and B meet at the point C. Find the x-coordinate of C. [5]

(ii) Show by integration that the area of the shaded region R_1 is the same as the area of the shaded region R_2. [4]

8 The equation of a curve is $y = \frac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

(iii) The region between the curve, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Show that the volume obtained is $\frac{12}{5}\pi$. [5]

Oct/Nov 2007

2 Find the area of the region enclosed by the curve $y = 2\sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$. [4]

9 A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q. Find

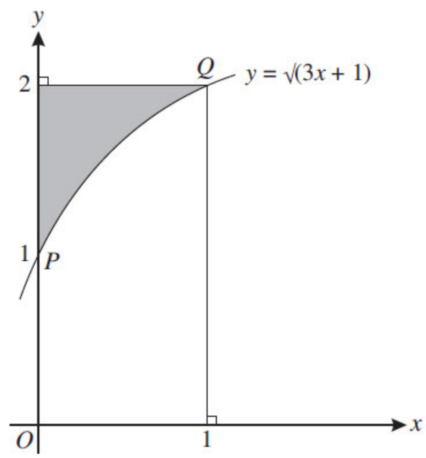
(i) the equation of the curve, [3]

(ii) the equation of the normal to the curve at P, [3]

(iii) the coordinates of Q. [3]

Oct/Nov 2008

9



The diagram shows the curve $y = \sqrt{(3x + 1)}$ and the points $P(0, 1)$ and $Q(1, 2)$ on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 2$.

(i) Find the area of the shaded region. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

Tangents are drawn to the curve at the points P and Q.

(iii) Find the acute angle, in degrees correct to 1 decimal place, between the two tangents. [4]

Oct/Nov 2009/11

4 The equation of a curve is $y = x^4 + 4x + 9$.

(i) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [3]

6 A curve is such that $\frac{dy}{dx} = k - 2x$, where k is a constant.

(i) Given that the tangents to the curve at the points where $x = 2$ and $x = 3$ are perpendicular, find the value of k. [4]

(ii) Given also that the curve passes through the point $(4, 9)$, find the equation of the curve. [3]

Oct/Nov 2009/12

1 The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$. Given that the curve passes through the point $(4, 6)$, find the equation of the curve. [4]

8 The function f is such that $f(x) = \dfrac{3}{2x+5}$ for $x \in \mathbb{R}, x \neq -2.5$.

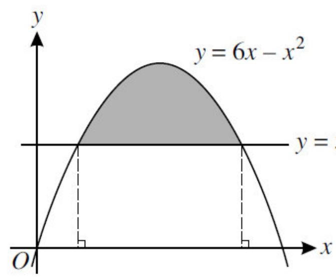
(i) Obtain an expression for $f'(x)$ and explain why f is a decreasing function. [3]

(ii) Obtain an expression for $f^{-1}(x)$. [2]

(iii) A curve has the equation $y = f(x)$. Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 2$ is rotated through 360° about the x-axis. [4]

May/June 2010/11

4



The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]

6 A curve is such that $\dfrac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point (9, 2) lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

May/June 2010/12

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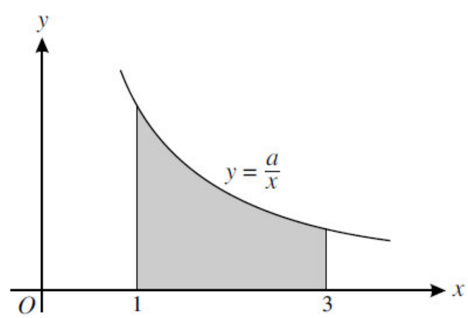

The diagram shows part of the curve $y = \dfrac{a}{x}$, where a is a positive constant. Given that the volume obtained when the shaded region is rotated through 360° about the x-axis is 24π, find the value of a. [4]

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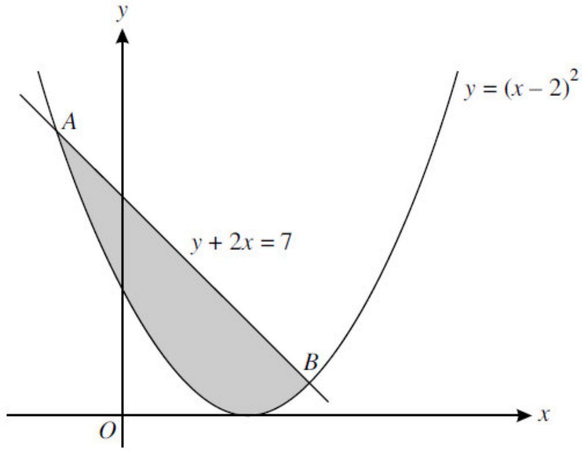

The diagram shows the curve $y = (x-2)^2$ and the line $y + 2x = 7$, which intersect at points A and B. Find the area of the shaded region. [8]

May/June 2010/13

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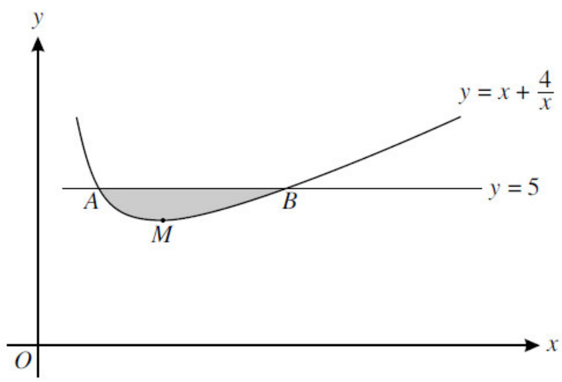

The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at M. The line $y = 5$ intersects the curve at the points A and B.

(i) Find the coordinates of A, B and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Oct/Nov 2010/11

1 Find $\int \left(x + \frac{1}{x}\right)^2 \mathrm{d}x$. [3]

11 The equation of a curve is $y = \frac{9}{2-x}$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through 360° about the x-axis. [4]

(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]

Oct/Nov 2010/12

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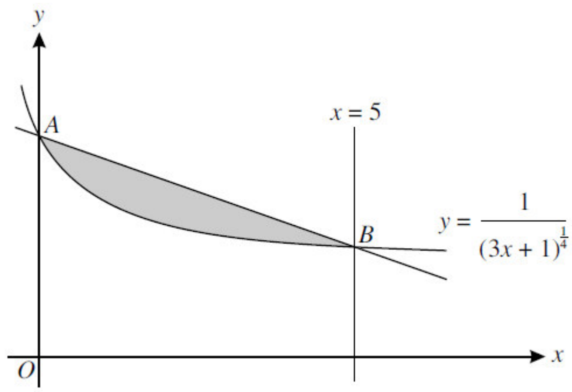


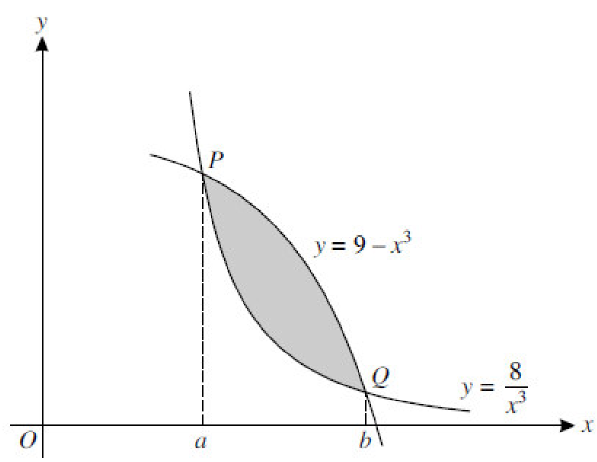
The diagram shows part of the curve $y = \frac{1}{(3x+1)^{\frac{1}{4}}}$. The curve cuts the y-axis at A and the line $x = 5$ at B.

(i) Show that the equation of the line AB is $y = -\frac{1}{10}x + 1$. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [9]

Oct/Nov 2010/13

11



The diagram shows parts of the curves $y = 9 - x^3$ and $y = \frac{8}{x^3}$ and their points of intersection P and Q. The x-coordinates of P and Q are a and b respectively.

(i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of a and the value of b. [4]

(ii) Find the area of the shaded region between the two curves. [5]

(iii) The tangents to the two curves at $x = c$ (where $a < c < b$) are parallel to each other. Find the value of c. [4]