Q1.

5 The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b. [2]

(ii) State the greatest and least values of $f(x)$. [2]

(iii) Solve the equation $f(x) + 1 = 0$. [3]

Q2.

9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x-b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $f(x) < 21$. [3]

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

Q3.

3 The function $f : x \mapsto a + b\cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that $f(0) = 10$ and that $f\left(\frac{2}{3}\pi\right) = 1$, find

(i) the values of a and b, [2]

(ii) the range of f, [1]

(iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]

Q4.

10 The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x. [3]

Q5.

11 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 1,$$
$$\mathrm{g} : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $\mathrm{fg}(x)$ and $\mathrm{gf}(x)$. [2]

(ii) Hence find the value of a for which $\mathrm{fg}(a) = \mathrm{gf}(a)$. [3]

(iii) Find the value of b $(b \neq a)$ for which $\mathrm{g}(b) = b$. [2]

(iv) Find and simplify an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$. [2]

The function h is defined by

$$\mathrm{h} : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $\mathrm{h}^{-1}(x)$. [2]

Q6.

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate $\mathrm{fg}(2)$. [2]

(ii) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $\mathrm{g}'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

Q7.

8 The function $\mathrm{f} : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

(i) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

(ii) State the range of f in terms of k. [1]

(iii) State the smallest value of p for which f is one-one. [1]

(iv) For the value of p found in part **(iii)**, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}, giving your answers in terms of k. [4]

Q8.

11 The function f is such that $f(x) = 8 - (x - 2)^2$, for $x \in \mathbb{R}$.

(i) Find the coordinates and the nature of the stationary point on the curve $y = f(x)$. [3]

The function g is such that $g(x) = 8 - (x - 2)^2$, for $k \leqslant x \leqslant 4$, where k is a constant.

(ii) State the smallest value of k for which g has an inverse. [1]

For this value of k,

(iii) find an expression for $g^{-1}(x)$, [3]

(iv) sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$. [3]

Q9.

8 **(i)** Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. [1]

The value of k is now given to be 7.

(iii) Find the range of f. [1]

(iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [5]

Q10.

10 The function f is defined by $f : x \mapsto 2x + k$, $x \in \mathbb{R}$, where k is a constant.

(i) In the case where $k = 3$, solve the equation $ff(x) = 25$. [2]

The function g is defined by $g : x \mapsto x^2 - 6x + 8$, $x \in \mathbb{R}$.

(ii) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions. [3]

The function h is defined by $h : x \mapsto x^2 - 6x + 8$, $x > 3$.

(iii) Find an expression for $h^{-1}(x)$. [4]

Q11.

10 Functions f and g are defined by

$$f : x \mapsto 2x + 1, \quad x \in \mathbb{R}, \quad x > 0,$$
$$g : x \mapsto \frac{2x - 1}{x + 3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(i) Solve the equation $gf(x) = x$. [3]

(ii) Express $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

(iii) Show that the equation $g^{-1}(x) = x$ has no solutions. [3]

(iv) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

Q12.

3 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 3,$$
$$g : x \mapsto x^2 - 2x.$$

Express $gf(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [5]

Q13.

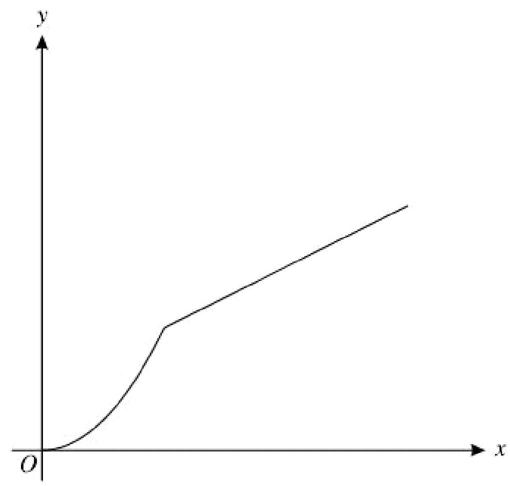
6 A curve has equation $y = f(x)$. It is given that $f'(x) = 3x^2 + 2x - 5$.

(i) Find the set of values of x for which f is an increasing function. [3]

(ii) Given that the curve passes through $(1, 3)$, find $f(x)$. [4]

Q14.

7



The diagram shows the function f defined for $0 \leqslant x \leqslant 6$ by

$$x \mapsto \tfrac{1}{2}x^2 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$x \mapsto \tfrac{1}{2}x + 1 \quad \text{for } 2 < x \leqslant 6.$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(iii) Obtain expressions to define $\mathrm{f}^{-1}(x)$, giving the set of values of x for which each expression is valid. [4]

Q15.

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$\mathrm{g} : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

(i) Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) State the domain of f^{-1}. [1]

(iv) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{g}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [4]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

Q16.

9 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 3 \quad \text{for } x \leqslant 0,$$
$$\mathrm{g} : x \mapsto x^2 - 6x \quad \text{for } x \leqslant 3.$$

(i) Express $\mathrm{f}^{-1}(x)$ in terms of x and solve the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

(ii) On the same diagram sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing the coordinates of their point of intersection and the relationship between the graphs. [3]

(iii) Find the set of values of x which satisfy $\mathrm{gf}(x) \leqslant 16$. [5]

Q17.

10 The function f is defined by $\mathrm{f}(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

(i) Express $\mathrm{f}(x)$ in the form $a(x-b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = \mathrm{f}(x)$. [4]

The function g is defined by $\mathrm{g}(x) = 4x^2 - 24x + 11$, for $x \leqslant 1$.

(ii) State the range of g. [2]

(iii) Find an expression for $\mathrm{g}^{-1}(x)$ and state the domain of g^{-1}. [4]

Q18.

6 The functions f and g are defined for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$ by

$$\mathrm{f}(x) = \tfrac{1}{2}x + \tfrac{1}{6}\pi,$$
$$\mathrm{g}(x) = \cos x.$$

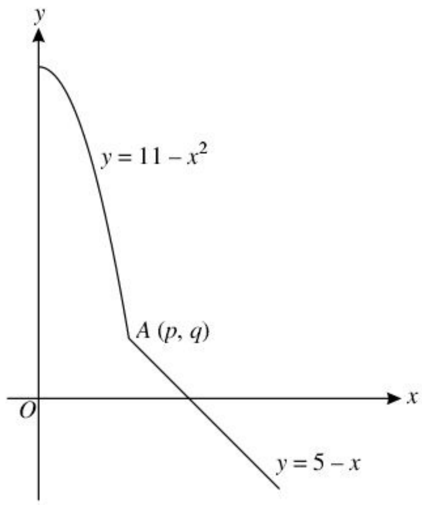
Solve the following equations for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) $\mathrm{gf}(x) = 1$, giving your answer in terms of π. [2]

(ii) $\mathrm{fg}(x) = 1$, giving your answers correct to 2 decimal places. [4]

Q19.

7



(i) The diagram shows part of the curve $y = 11 - x^2$ and part of the straight line $y = 5 - x$ meeting at the point $A\ (p, q)$, where p and q are positive constants. Find the values of p and q. [3]

(ii) The function f is defined for the domain $x \geqslant 0$ by

$$f(x) = \begin{cases} 11 - x^2 & \text{for } 0 \leqslant x \leqslant p, \\ 5 - x & \text{for } x > p. \end{cases}$$

Express $f^{-1}(x)$ in a similar way. [5]

Q20.

5 The function f is defined by

$$f : x \mapsto x^2 + 1 \quad \text{for} \quad x \geqslant 0.$$

(i) Define in a similar way the inverse function f^{-1}. [3]

(ii) Solve the equation $ff(x) = \frac{185}{16}$. [3]

Q21.

10 The function f is defined by $f : x \mapsto x^2 + 4x$ for $x \geqslant c$, where c is a constant. It is given that f is a one-one function.

(i) State the range of f in terms of c and find the smallest possible value of c. [3]

The function g is defined by $g : x \mapsto ax + b$ for $x \geqslant 0$, where a and b are positive constants. It is given that, when $c = 0$, $gf(1) = 11$ and $fg(1) = 21$.

(ii) Write down two equations in a and b and solve them to find the values of a and b. [6]