Q1.

4	$y - 6x - x^2$		
	Meets $y = 5$ when $x = 1$ or $x = 5$.	B1	co
	Integral = $3x^2 - \frac{1}{3}x^3$	M1 A1	attempt to integrate. co.
	Their limits (1 to 5) used → 30⅔	DM1	value at top limit − value at lower
	Area of rectangle = 20	B1√	co to his x values
	Shaded area = 10⅔	A1	co
		[6]	
	(integral of $6x - x^2 - 5$ B1, M1, A1 DM1 as above, then "$-5x$" B1√ A1)		

Q2.

5	$\frac{dy}{dx} - \frac{6}{\sqrt{3x-2}}$		
	(i) $x = 2$, tangent has gradient 3	M1	Use of $m_1m_2 = -1$ with dy/dx
	→ normal has gradient $-\frac{1}{3}$	M1 A1	Correct form of line eqn. for normal
	→ $y - 11 = -\frac{1}{3}(x-2)$	[3]	
	(ii) Integrate → $6\frac{\sqrt{3x-2}}{\frac{1}{2}} \div 3$	B1 B1	Without the ÷3 For ÷3, even if B0 above
	→ $y - 4\sqrt{3x-2} + c$ through (2,11)	M1	Using (2, 11) for c
	→ $y - 4\sqrt{3x-2} + 3$	A1	co
		[4]	

Q3.

9	$y = x + \frac{4}{x}$		
	(i) $x + \frac{4}{x} - 5 \rightarrow A(1, 5), B(4, 5)$	B1 B1	co. co.
	$\frac{dy}{dx} = 1 - \frac{4}{x^2}$	M1	Differentiates.
	$= 0$ when $x = 2$, $M(2, 4)$.	DM1 A1 [5]	Setting to 0. co.
	(ii) Vol of cylinder $= \pi 5^2.3$	B1	Any valid method.
	Vol under curve $= \pi \int y^2 \, dx$	M1	Attempt at integrating y^2
	Integral $= \frac{x^3}{3} - \frac{16}{x} + 8x$	A2, 1, 0	Allow if no π present.
	Uses his limits "1 to 4" $\rightarrow 75\pi - 57\pi - 18\pi$	DM1 A1 [6]	Using his limits. co.

Q4.

3	(i) Correct shape – touching positive x-axis	B1 [1]	Ignore intersections with axes
	(ii) $(\pi) \int (x-2)^4 \, dx$	M1	Use $(\pi) \int y^2 dx$ & attempt integrate but expansion before integn needs 5 terms
	$(\pi)\left[\frac{(x-2)^5}{5}\right]$	A1	
	$(\pi)[0 - (-32)/5)]$	M1	Use of limits 0, 2 on *their* $(\pi) \int y^2 dx$
	$\frac{32\pi}{5}$ or 6.4π	A1 [4]	cao Rotation about y-axis max 1/5

Q5.

7	(i) $\frac{3(1+2x)^{-1}}{-1} + (c)$	B1	
	$y = \frac{3(1+2x)^{-1}}{-2} + (c)$	B1(indep)	Division by 2 $y =$ necessary
	Sub (1, (1/2))	M1	Dependent on c present
	$\frac{1}{2} - \frac{3}{-6} + c \Rightarrow c = -1$	A1 [4]	Use of $y = mx + c$ etc. gets 0/4
	(ii) $(1 + 2x)^2 (>) 9$ or $4x^2 + 4x - 8 (>) 0$ **OE**	M1	
	$1, -2$	A1	
	$x > 1, x < -2$ ISW	A1 [3]	

Q6.

9 $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ $P(9, 5)$		
(i) $y = 4\sqrt{x} - x (+c)$ Uses (9, 5) in an integrated expression $\rightarrow c = 2$	B1 B1 M1 A1 [4]	Ignore + c. Substitution of point after integration. co.
(ii) $\frac{dy}{dx} = 0 \rightarrow x = 4, y = 6$	M1 A1 A1 [3]	Attempt to solve $dy/dx = 0$. x correct. y correct.
(iii) $\frac{d^2y}{d^2x} = -x^{\frac{-3}{2}} \rightarrow$ –ve $\rightarrow$ Max	B1 B1√ [2]	co. √ for correct deduction.
(iv) $\frac{dy}{dx} = -\frac{1}{3}$ Perpendicular $m = 3$ $\tan\theta = 3$ Angle is $\tan^{-1}3$ $k = 3$	M1 A1 [2]	Use of $m_1m_2 = -1$ Needs $k = 3$

Q7.

11 (i) $x = \frac{4}{y^2} - 1$	**B1** **[1]**	**AG** At least 1 step of working needed
(ii) $\int\left(\frac{4}{y^2} - 1\right)dy = \left[-\frac{4}{y} - y\right]$	**B1B1**	
Upper limit = 2 $\left[\left(-\frac{4}{2} - 2\right) - (-4 - 1)\right]$ 1	**B1** **M1** **A1** **[5]**	For $-\frac{4}{y}, -y$ Apply limits 1 and *their* 2 'correctly' SC B2 for $\int 2(x+1)^{-\frac{1}{2}}\,dx - 3 \rightarrow 1$
(iii) $(\pi)\int x^2\,dy - (\pi)\int\left(\frac{16}{y^4} - \frac{8}{y^2} + 1\right)dy$	**B1B1**	
$(\pi)\left[\frac{-16}{3y^3} + \frac{8}{y} + y\right]$	**B1**	
$(\pi)\left[\left(\frac{-16}{24} + 4 + 2\right) - \left(\frac{-16}{3} + 8 + 1\right)\right]$	**M1**	Apply limits 1 and *their* 2 'correctly'
$\frac{5\pi}{3}$	**A1** **[5]**	

Q8.

5 $x - \frac{8}{y^2} - 2$; at $x = 0, y = 2$	B1	co
$\rightarrow x^2 - \frac{64}{y^4} - \frac{32}{y^2} + 4$		
Integral of $x^2 = \frac{64y^{-3}}{-3} - \frac{32y^{-1}}{-1} + 4y$	B1B1B1	All co.
Uses limits 1 to 2	M1	Uses 1 to 2 or 2 to 1.
$\rightarrow 6\frac{2}{3}\pi$	A1 [6]	co.

Q9.

9 $\frac{d^2y}{dx^2} = -4x$		
(i) $\frac{dy}{dx} = -2x^2 + c$	B1	For $-2x^2$
$\frac{dy}{dx} = 0$ when $x = 2$, $\rightarrow c = 8$	B1	$c = 8$
$y = -\frac{2x^3}{3} + 8x \ (+C)$	B1 B1√	For each term – √ on "c"– ignore $(+C)$
Subs (2, 12) $\rightarrow C = \frac{4}{3}$	M1 A1 [6]	Uses (2, 12) to find C.
(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$	M1	
$= -10 \times 0.05$		Must use. Enough to see product of
$\rightarrow$ decreasing at 0.5 units per second	A1 [2]	gradient and rate. bod over notation.

Q10.

10 (i) $\frac{dy}{dx} - 4(x-2)^3$	**B1**	Or $4x^3 - 24x^2 + 48x - 32$
Grad of tangent = –4	**M1**	Sub $x = 1$ into *their* derivative
Eq. of tangent is $y - 1 = -4(x - 1)$	**M1**	Line thru (1, 1) and with m from deriv
$\rightarrow B(\frac{5}{4}, 0)$	**A1**	
Grad of normal = $\frac{1}{4}$	**M1**	Use of $m_1 m_2 = -1$
Eq. of normal is $y - 1 = \frac{1}{4}(x-1) \rightarrow C(0, \frac{3}{4})$	**A1** [6]	

(ii) $AC^2 = 1^2 + \left(\frac{1}{4}\right)^2$	M1		
$\frac{\sqrt{17}}{4}$	A1 [2]		Allow $\sqrt{\frac{17}{16}}$
(iii) $\int (x-2)^4 dx = \frac{(x-2)^5}{5}$	B1		Or $\frac{x^5}{5} - 2x^4 + 8x^3 - 16x^2 + 16x$
$\left[0 - (-\frac{1}{5})\right] - \frac{1}{5}$	M1		Apply limits $1 \rightarrow 2$ for curve
$\Delta - \frac{1}{2} \times 1 \times (their\ \frac{5}{4} - 1) - \frac{1}{8}$	M1		Or $\int_1^{\frac{5}{4}} (-4x+5)\, dx = \frac{1}{8}$
$\frac{1}{5} - \frac{1}{8} - \frac{3}{40}$ or 0.075	A1 [4]		

Q11.

1	$\frac{dy}{dx} = \sqrt{2x+5}$			
	$\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \div 2 \quad (+c)$	B1 B1		B1 Everything without "÷2". B1 "÷2"
	Uses (2, 5) $\rightarrow$ $c = -4$	M1 A1	[4]	Uses point in an integral.

Q12.

11	$y = \frac{8}{\sqrt{x}} - x$			
(i)	$\frac{dy}{dx} = -4x^{-\frac{3}{2}} - 1$	B1		needs both
	$= -\frac{3}{2}$ when $x = 4$.	M1		Subs $x = 4$ into dy/dx
	Eqn of BC $y - 0 = -\frac{3}{2}(x-4)$	M1 A1		Must be using differential + correct form of line at $B(4,0)$.
	$\rightarrow C\ (1, 4\frac{1}{2})$		[4]	
(ii)	area under curve = $\int (\frac{8}{\sqrt{x}} - x)$			
	$= \frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2$	B1 B1		(both unsimplified)
	Limits 1 to 4 $\rightarrow$ $8\frac{1}{2}$	M1		Using correct limits.
	Area under tangent = $\frac{1}{2} \times 4\frac{1}{2} \times 3 = 6\frac{3}{4}$	M1		Or could use calculus)
	Shaded area = $1\frac{3}{4}$	A1	[5]	

Q13.

6	$\frac{dy}{dx} = k - 2x$			
	(i) At $x = 2, m = (k-4)$ $x = 3$ $m = (k-6)$ $(k-4)(k-6) = -1$ $\rightarrow k = 5$	M1 M1 DM1A1 [4]		Obtains either gradient as f(k). Uses $m_1 m_2 = -1$ with gradients f(k) Soln of quadratic = 0. co (watch for fortuitous answers)
	(ii) $y = kx - x^2 (+ c)$ Substitutes (4, 9) $\rightarrow c = 5$	B1√ M1 A1 [3]		For integration without c Realises need to substitute for x and y co (nb If $k = 5$ is fortuitous, loses last A1)

Q14.

1	$\int\left(x + \frac{1}{x}\right)^2 dx$ $= \frac{x^3}{3} - \frac{1}{x} + 2x + (c)$	B1 × 3	[3]	co. Omission of middle term of expansion can still get 2/3.

Q15.

11	$y = \frac{9}{2-x}$			
	(i) $\frac{dy}{dx} = -9(2-x)^{-2} \times -1$	B1 B1		Without the "× −1" Indep. With the "×−1". Indep.
	$\frac{9}{(2-x)^2} \neq 0$. No turning points.	B1√	[3]	√ provided of form $k \div (2-x)^2$.
	(ii) $V = \pi \int \frac{81}{(2-x)^2} dx$			
	$\int y^2 dx = -81(2-x)^{-1} \div (-1)$	B1 B1		Answer without the "÷ −1 including π For "÷ −1".
	Use of limits 0 to 1	M1		Uses both limits in an integral of y^2 – if "0" ignored, M0.
	$\rightarrow \frac{81\pi}{2}$ (or 127)	A1	[4]	co (If π omitted – max 3/4)
	(iii) $\frac{9}{2-x} - x + k$	M1		Elimination of y
	$\rightarrow x^2 - 2x + kx - 2k + 9 = 0$ Uses $b - 4ac$	M1		Uses discriminant
	$\rightarrow k^2 + 4k - 32$ $\rightarrow$ end-points of 4 and −8	A1		End-values correct.
	Range for 2 points of intersection $\rightarrow k < -8$, $k > 4$.	A1	[4]	Accept <, >.

Q16.

11	**(i)** $9-x^3-\frac{8}{x^3}$	M1		Together with attempt to mult by x^3
	$x^6-9x^3+8=0$	A1		**AG** completely correct working
	$(X-1)(X-8)=0 \rightarrow X=1$ or 8	M1		Attempt to solve quadratic in X or x^3
	$a=1, b=2$	A1	[4]	
	(ii) $\int_1^2\left[\left(9-x^3\right)-\frac{8}{x^3}\right]dx$	M1		Intention to integrate the difference y_1-y_2 not $\pi(y_1-y_2)$
	$\left[9x-\frac{x^4}{4}\right].\left[\frac{-4}{x^2}\right]$	B1 B1		
	$18-4+1-(9-\frac{1}{4}+4)$	M1		Correct use of their limits once
	$2\frac{1}{4}$	A1	[5]	
	(iii) $\frac{dy}{dx}=\frac{-24}{x^4}$, $\frac{dy}{dx}=-3x^2$	B1, B1		cao
	$\frac{-24}{c^4}=-3c^2$			
	$c^6=8$	M1		Equating and solution
	$c-\sqrt{2}$ or $8^{1/6}$ or 1.41(4...)	A1	[4]	Accept x or c

Q17.

10	**(i)** B $-(0,1)$ C $-(4,3)$	B1, B1	[2]	If BOBO then SCB1 for both $y=1$ & $x=4$
	(ii) $\frac{\delta y}{\delta x}=\frac{1}{2}\times 2(1+2x)^{-\frac{1}{2}}$	M1A1		$-\frac{1}{2}$ required & at least one of $\frac{1}{2}\times 2$ for M1
	Grad. of normal $=-3$	B1		
	$y-3=-3(x-4)$ or $y=-3x+15$ oe	B1√	[4]	Ft only from *their* C
	(iii) $y^2=1+2x \Rightarrow x=\frac{1}{2(y^2-1)}$ SOI	B1		$\int x^2\delta y$, square $\frac{1}{2}(y^2-1)$ & attempt int^n
	$(\pi)\times\frac{1}{4}\times\int(y^4-2y^2+1)\delta y$	M1		
	$(\pi)\times\frac{1}{4}\left[\frac{y^5}{5}-\frac{2y^3}{3}+y\right]$	A1		Apply limits $0 \rightarrow$ *their* 1 (from *their B*)
	$(\pi)\times\frac{1}{4}\left[\frac{1}{5}-\frac{2}{3}+1\right]$	DM1		cao SCB1 for $\int y^2\delta x \rightarrow \frac{\pi}{4}$ (scores 1/5)
	$\frac{2}{15}\pi$	A1	[5]	

Q18.

10 (i)	$\int(x+1)^{\frac{1}{2}}-(x+1)$ or $\int(y^2-1)-(y-1)$	M1	Dealing with line as a triangle or integral with correct limits.
	$\frac{2}{3}(x+1)^{\frac{3}{2}}-\frac{1}{2}x^2-x$ or $\frac{1}{3}y^2-\frac{1}{2}y^2$	M1A1	Attempt at integral of curve.
	$\frac{2}{3}-\left(0-\frac{1}{2}+1\right)$ or $\frac{1}{3}-\frac{1}{2}$	DM1	Applying limits $-1\rightarrow 0$ or $0\rightarrow 1$ to curve
	$\frac{1}{6}$	A1 [5]	π included loses last mark.
(ii)	$V_1=(\pi)\int(y^2-1)^2=(\pi)\int y^4-2y^2+1$	M1	
	$(\pi)\left[\frac{y^5}{5}-\frac{2y^2}{3}+y\right]$	A1	Attempt at $\int x^2$ dy for curve
	$(\pi)\left[\frac{1}{5}-\frac{2}{3}+1\right]$	DM1	Apply limits $0\rightarrow 1$
	$V_1=\frac{8}{15(\pi)}$ or $0.533(\pi)$ (AWRT)	A1	
or	$(\pi)\left[y\uparrow 3/3-y\uparrow 2+y\right]$	M1	
	$V_2=\frac{1}{3}\pi$	A1	Vol of cone or attempt to $\int x^2dy$ for line
	Volume $=\frac{8}{15}\pi\ \frac{1}{-3}\pi=\frac{1}{5}\pi$ (or 0.628)	A1 [7]	
OR	$(y^4-2y^2+1)-(y^2-2y+1)$	M1	
	$(\pi)\int y^4-3y^2+2y$	M1	
	$(\pi)\left[y\uparrow 5/5-y\uparrow 3+y\uparrow 2\right]$	A1,A1,A1	Attempt to $\int x^2dy$
	$(\pi)\left[\frac{1}{5}-1+1\right]$	DM1	Attempt to $\int (x_1{}^2-x_2{}^2)$
	$\frac{1}{5}\pi$	A1	Apply limits $0\rightarrow 1$ dependent on first M1

Q19.

2	$y=\frac{4}{x^2}-x\ (+c)$ Sub (2, 4) $\rightarrow c=5$	M1A1 DM1A1 [4]	Attempt integration. cao Dependent on c present

Q20.

8	**(i)** $y^2 = 3y \Rightarrow y(y-3) = 0 \Rightarrow y = 3$ (or 0) $x = \frac{1}{2}$ or 5 $(\Rightarrow a = 5)$ **AG**	**M1** **A1** [2]	**OR** form equation in x and attempt solution **OR** sub x=5 each eq (M1) → y = 3(twice) (A1) (5,3) subst only once scores 0/2
	(ii) $\left[\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷ 2], $\left[\frac{2}{3}\times\frac{x^2}{2}-\frac{x}{3}\right]$ $\left[\frac{27}{3}-0\right]$, $\left[\frac{25}{3}-\frac{5}{3}-\left(\frac{1}{12}-\frac{1}{6}\right)\right]$ Subtract areas at some stage $\frac{9}{4}$ oe	**B1B1B1** **M1** **M1** **A1** [6]	Or $\Delta = \frac{1}{2}(5-\frac{1}{2})\times 3$ Apply limits ½ and 5 for, at least, curve Dependent on some integration cao 9/4 with no working scores 0/6, but 9 − 27/4 = 9/4 scores 1/6 (M1 subtraction)

Q21.

11	**(i)** $a = 2$	B1 [1]	
	(ii) $y =$	B1 B2, 1, 0 √	–1 for eeoo
	$b = \frac{2}{3}$	B1 [4] B2, 1, 0	cao –1 for eeoo
	(iii) area $= \int y\,dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]$ $\frac{4}{3}$	M1 A1 [4]	Apply limits 0→2 – ft *their a* from **(i)** cao
	(iv) $\frac{d^2y}{dx^2} = 6x - 8 = 0 \Rightarrow x = \frac{4}{3}$ When $x = \frac{4}{3}$, $\frac{dy}{dx}$ (or m) =	M1 A1 DM1 A1 [4]	Attempt $\frac{d^2y}{dx^2}$ and set = 0 cao

Q22.

2	Attempt integration $f(x) - 2(x+6)^{\frac{1}{2}} - \frac{6}{x}(+c)$	**M1** **A1A1**	Accept unsimplified terms
	$2(3) - \frac{6}{3} + c - 1$ $c = -3$	**M1** **A1** [5]	Sub.x = 3, y = 1. c must be present

Q23.

10 **(i)**	$\frac{dy}{dx} - [3(3-2x)^2] \times [-2]$	**B1B1**	OR $-54+72x-24x^2$ B2,1,0
	At $x = \frac{1}{2}$, $\frac{dy}{dx} = -24$	**M1**	
	$y-8 = -24\left(x-\frac{1}{2}\right)$	**DM1**	
	$y = -24x+20$	**A1** [5]	
(ii)	Area under curve $= \left[\frac{(3-2x)^4}{4}\right] \times \left[-\frac{1}{2}\right]$	**B1B1**	OR $27x-27x^2+12x^3-2x^4$ B2,1,0
	$-2-\left(-\frac{81}{8}\right)$	**M1**	Limits 0→ ½ applied to integral with intention of subtraction shown
	Area under tangent $= \int(-24x+20)$	**M1**	or area trap =½(20 + 8) × ½
	$-\lvert -12x^2+20x \rvert$ or 7 (from trap)	**A1**	Could be implied
	$\frac{9}{8}$ or 1.125	**A1** [6]	Dep on both M marks

Q24.

2	$f(x) = 2x^{-\frac{1}{2}} + x\ (+c)$	M1A1	Attempt integ $x^{-\frac{1}{2}}$ or + x needed for M
	$5 = -2 \times \frac{1}{2} + 4 + c$	M1	Sub (4, 5). c must be present
	$c = 2$	A1 [4]	

Q25.

11 **(i)**	$\frac{dy}{dx} - \left[\frac{1}{2}(x4+4x+4)^{-\frac{1}{2}}\right] \times [4x^3+4]$	B1B1	
	At $x = 0$, $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2} \times 4 - (1)$	M1	Sub $x = 0$ and attempt eqn of line following differentiation.
	Equation is $y - 2 = x$	A1 [4]	
(ii)	$x+2 = \sqrt{x^4+4x+4} \Rightarrow (x+2)^2$	B1	**AG** www
	$= x4+4x+4$		
	$x^2 - x^4 = 0$ oe	B1	
	$x = 0, \pm 1$	B2,1,0 [4]	
(iii)	$(\pi)\left[\frac{x^5}{5}+2x^2+4x\right]$	M1A1	Attempt to integrate y^2
	$(\pi)\left[0-\left(\frac{-1}{5}+2-4\right)\right]$	DM1	
	$\frac{11\pi}{5}$ (6.91) oe	A1 [4]	Apply limits $-1 \rightarrow 0$