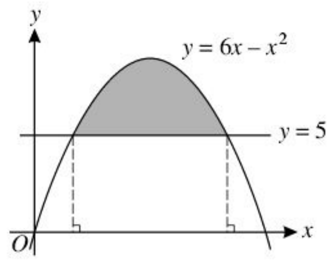
Q1.

4



The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]

Q2.

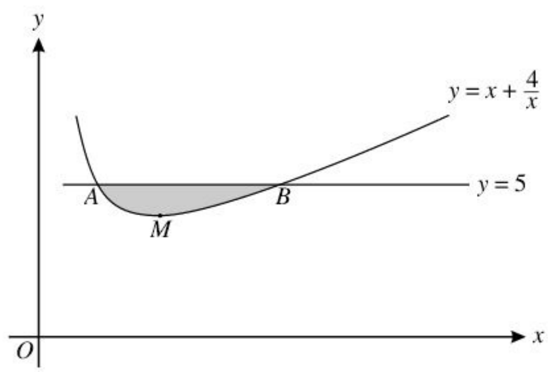
5 The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(3x-2)}}$. Given that the curve passes through the point $P(2, 11)$, find

(i) the equation of the normal to the curve at P, [3]

(ii) the equation of the curve. [4]

Q3.

9



The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at M. The line $y = 5$ intersects the curve at the points A and B.

(i) Find the coordinates of A, B and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Q4.

3 **(i)** Sketch the curve $y = (x - 2)^2$. [1]

(ii) The region enclosed by the curve, the x-axis and the y-axis is rotated through 360° about the x-axis. Find the volume obtained, giving your answer in terms of π. [4]

Q5.

7 A curve is such that $\frac{dy}{dx} = \frac{3}{(1+2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]

Q6.

9 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ and $P\ (9, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

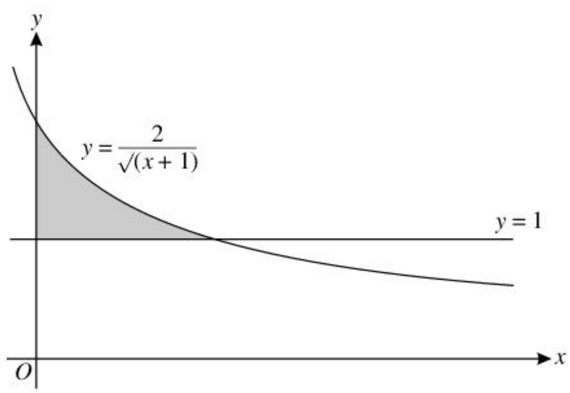
(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\frac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1} k$ with the positive x-axis. Find the value of k. [2]

Q7.

11



The diagram shows the line $y = 1$ and part of the curve $y = \frac{2}{\sqrt{(x+1)}}$.

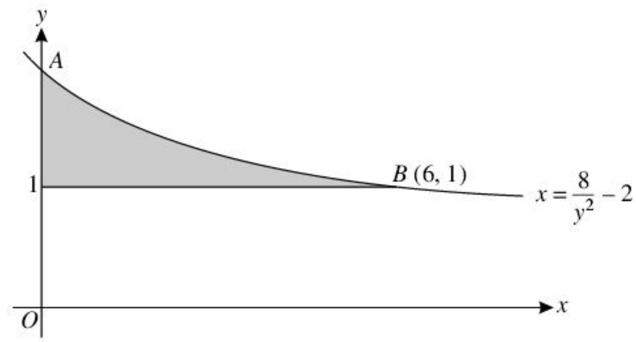
(i) Show that the equation $y = \frac{2}{\sqrt{(x+1)}}$ can be written in the form $x = \frac{4}{y^2} - 1$. [1]

(ii) Find $\int \left(\frac{4}{y^2} - 1\right) dy$. Hence find the area of the shaded region. [5]

(iii) The shaded region is rotated through 360° about the **y-axis**. Find the exact value of the volume of revolution obtained. [5]

Q8.

5



The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y-axis at the point A. The point $B\ (6,\ 1)$ lies on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the **y-axis**. [6]

Q9.

9 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at $(2,\ 12)$.

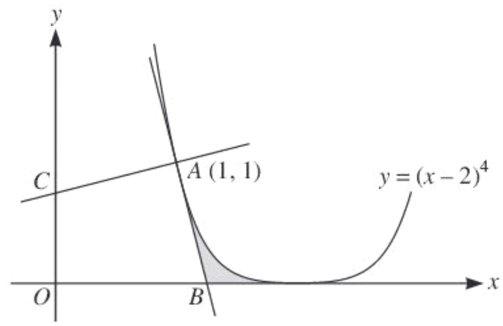
(i) Find the equation of the curve. [6]

A point P moves along the curve in such a way that the x-coordinate is increasing at 0.05 units per second.

(ii) Find the rate at which the y-coordinate is changing when $x = 3$, stating whether the y-coordinate is increasing or decreasing. [2]

Q10.

10



The diagram shows part of the curve $y = (x - 2)^4$ and the point $A\,(1,\ 1)$ on the curve. The tangent at A cuts the x-axis at B and the normal at A cuts the y-axis at C.

(i) Find the coordinates of B and C. [6]

(ii) Find the distance AC, giving your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers. [2]

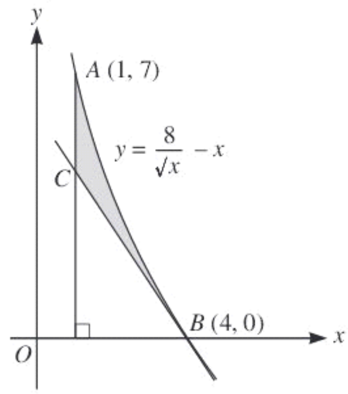
(iii) Find the area of the shaded region. [4]

Q11.

1 A curve is such that $\frac{dy}{dx} = \sqrt{(2x + 5)}$ and $(2,\ 5)$ is a point on the curve. Find the equation of the curve. [4]

Q12.

11



The diagram shows part of the curve $y = \frac{8}{\sqrt{x}} - x$ and points $A\,(1,\ 7)$ and $B\,(4,\ 0)$ which lie on the curve. The tangent to the curve at B intersects the line $x = 1$ at the point C.

(i) Find the coordinates of C. [4]

(ii) Find the area of the shaded region. [5]

Q13.

6 A curve is such that $\frac{dy}{dx} = k - 2x$, where k is a constant.

(i) Given that the tangents to the curve at the points where $x = 2$ and $x = 3$ are perpendicular, find the value of k. [4]

(ii) Given also that the curve passes through the point (4, 9), find the equation of the curve. [3]

Q14.

1 Find $\int \left(x + \frac{1}{x}\right)^2 dx$. [3]

Q15.

11 The equation of a curve is $y = \frac{9}{2 - x}$.

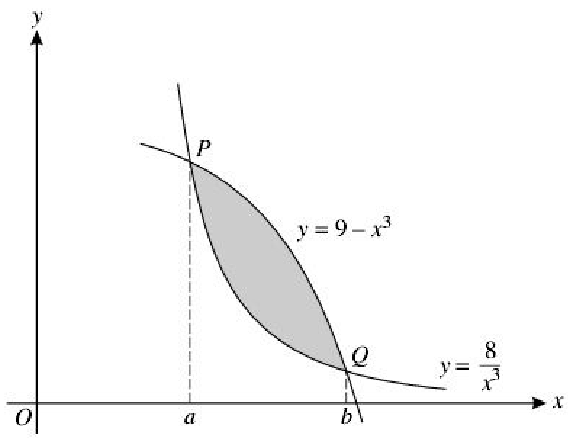
(i) Find an expression for $\frac{dy}{dx}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through 360° about the x-axis. [4]

(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]

Q16.

11



The diagram shows parts of the curves $y = 9 - x^3$ and $y = \frac{8}{x^3}$ and their points of intersection P and Q. The x-coordinates of P and Q are a and b respectively.

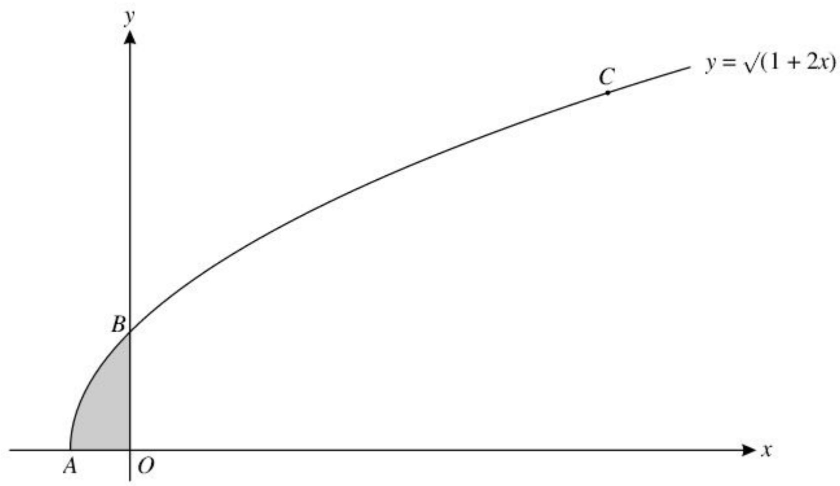
(i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of a and the value of b. [4]

(ii) Find the area of the shaded region between the two curves. [5]

(iii) The tangents to the two curves at $x = c$ (where $a < c < b$) are parallel to each other. Find the value of c. [4]

Q17.

10



The diagram shows the curve $y = \sqrt{(1 + 2x)}$ meeting the x-axis at A and the y-axis at B. The y-coordinate of the point C on the curve is 3.

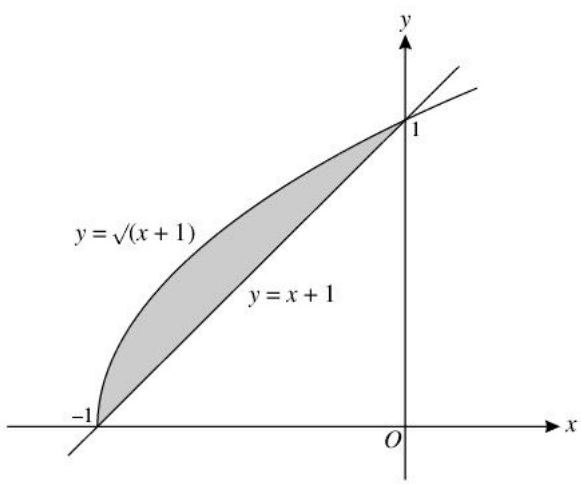
(i) Find the coordinates of B and C. [2]

(ii) Find the equation of the normal to the curve at C. [4]

(iii) Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [5]

Q18.

10



The diagram shows the line $y = x + 1$ and the curve $y = \sqrt{(x + 1)}$, meeting at $(-1, 0)$ and $(0, 1)$.

(i) Find the area of the shaded region. [5]

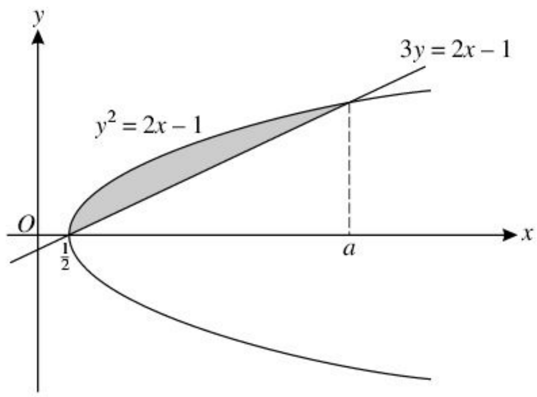
(ii) Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [7]

Q19.

2 A curve is such that $\frac{dy}{dx} = -\frac{8}{x^3} - 1$ and the point (2, 4) lies on the curve. Find the equation of the curve. [4]

Q20.

8



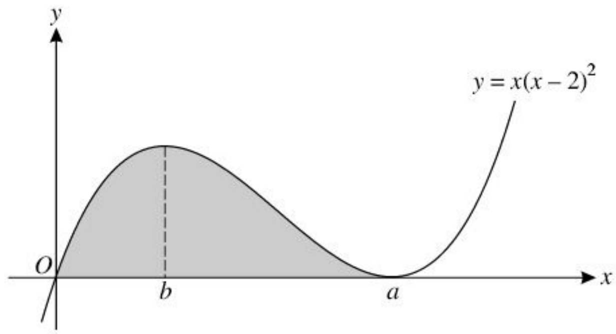
The diagram shows the curve $y^2 = 2x - 1$ and the straight line $3y = 2x - 1$. The curve and straight line intersect at $x = \frac{1}{2}$ and $x = a$, where a is a constant.

(i) Show that $a = 5$. [2]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

Q21.

11



The diagram shows the curve with equation $y = x(x-2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x-coordinate of the maximum point is b, where a and b are constants.

(i) State the value of a. [1]

(ii) Find the value of b. [4]

(iii) Find the area of the shaded region. [4]

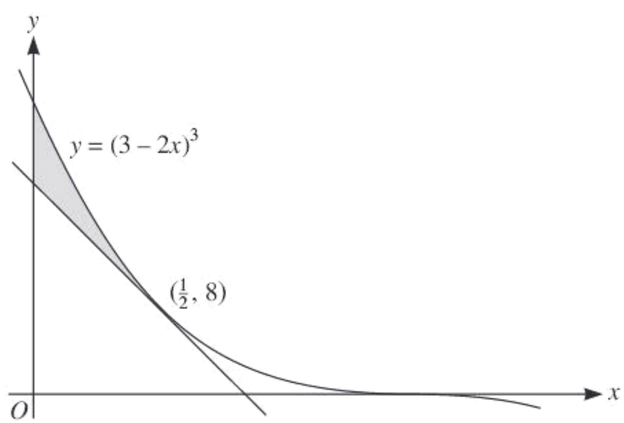
(iv) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m. Find the value of m. [4]

Q22.

2 A curve has equation $y = f(x)$. It is given that $f'(x) = \frac{1}{\sqrt{(x+6)}} + \frac{6}{x^2}$ and that $f(3) = 1$. Find $f(x)$. [5]

Q23.

10



The diagram shows the curve $y = (3-2x)^3$ and the tangent to the curve at the point $(\frac{1}{2}, 8)$.

(i) Find the equation of this tangent, giving your answer in the form $y = mx + c$. [5]

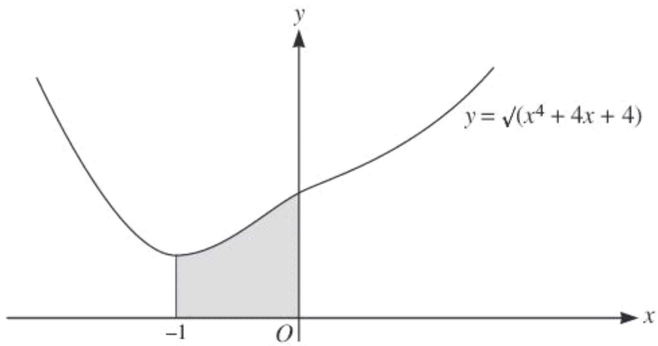
(ii) Find the area of the shaded region. [6]

Q24.

2 A curve has equation $y = f(x)$. It is given that $f'(x) = x^{-\frac{3}{2}} + 1$ and that $f(4) = 5$. Find $f(x)$. [4]

Q25.

11



The diagram shows the curve $y = \sqrt{(x^4 + 4x + 4)}$.

(i) Find the equation of the tangent to the curve at the point (0, 2). [4]

(ii) Show that the x-coordinates of the points of intersection of the line $y = x + 2$ and the curve are given by the equation $(x + 2)^2 = x^4 + 4x + 4$. Hence find these x-coordinates. [4]

(iii) The region shaded in the diagram is rotated through 360° about the x-axis. Find the volume of revolution. [4]