Q1.

1 $\tan x = k$		
(i) $\tan(\pi - x) = -k$	B1 [1]	co. www Mark final answers
(ii) $\tan\left(\frac{\pi}{2} - x\right) = \frac{1}{k}$	B1 [1]	co. www
(iii) $\sin x = \frac{k}{\sqrt{1+k^2}}$ from 90° triangle.	M1 A1 [2]	Any valid method – 90° triangle or formulae.

Q2.

4 **(i)** $2\sin x \tan x + 3 - 0$		
$2\sin x \frac{\sin x}{\cos x} + 3 - 0$	M1	For using tan = sin ÷ cos
$2\frac{(1-\cos^2 x)}{\cos x} + 3 - 0$	M1	For using $\sin^2 + \cos^2 = 1$ <u>and</u> everything correct
$\rightarrow 2\cos^2 x - 3\cos x - 2 - 0$	[2]	Answer given – check.
(ii) $2\cos^2 x - 3\cos x - 2 - 0$ $\rightarrow \cos x - -½$ or 2 $x = 120°$ or 240°	M1 A1 B1√ [3]	Solution of quadratic. co. √ for 360 – his answer.

Q3.

5 **(i)** $\frac{2\sin^2\theta\sin^2\theta}{1-\sin^2\theta} = 1$	M1	Equation as function of $\sin\theta$
$2\sin^4\theta + \sin^2\theta - 1 - 0$ **AG**	A1 [2]	
(ii) $(2\sin^2\theta - 1)(\sin^2\theta + 1) - 0$	M1	Or use formula on quadratic in $\sin^2\theta$
$\sin\theta - \frac{(\pm)1}{\sqrt{2}}$	A1	
$\theta = 45°, 135°$	A1	
$\theta = 225°, 315°$	A1 [4]	Provided no excess solutions in range

Q4.

8 **(i)** $\left(\frac{1}{\sin\theta}-\frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$		
$\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2 - \frac{(1-\cos\theta)^2}{\sin^2\theta}$	M1	Use of tan = sin/cos
$-\frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} - \frac{1-\cos\theta}{1+\cos\theta}$	M1 A1 [3]	Use of $\sin^2+\cos^2=1$. All correct. (NB ag. – ensure cancelling has been done)
(ii) $\left(\frac{1}{\sin\theta}-\frac{1}{\tan\theta}\right)^2 - \frac{2}{5}$		
$\frac{1-\cos\theta}{1+\cos\theta} - \frac{2}{5}$	M1	Uses part **(i)** to obtain an eqn in $\cos\theta$
$\cos\theta - \frac{3}{7}$	A1	co
$\theta = 64.6°$ or $295.4°$	A1 A1 √ [4]	co. √ for 360 – "1[st] answer".

Q5.

1 $\tan 2x = 2$ $2x = 63.4$ or 243.4 $x = 31.7$ or 121.7 (allow 122)	**M1** **A1** **A1A1**√ **[4]**	1 solution sufficient For 2[nd] A1 allow 90 + 1[st] soln prov. only 2 solns in range. Alt methods possible

Q6.

1 $\tan^2\theta - \sin^2\theta - \tan^2\theta\sin^2\theta$		
(i) $\frac{s^2}{c^2} - s^2$	M1	Use of $s \div c = t$
$\rightarrow \frac{s^2 - s^2c^2}{c^2} = \frac{s^2(1-c^2)}{c^2}$	M1	Use of $s^2 + c^2 = 1$
$\rightarrow t^2s^2$	A1 [3]	All ok
(ii) RHS > 0 $\rightarrow$ $\tan^2\theta > \sin^2\theta$ QED $\tan\theta > \sin\theta$ if θ acute.	B1 [1]	Realises RHS > 0

Q7.

4 $\sin 2x + 3\cos 2x - 0$		
(i) $\rightarrow$ $\tan 2x = -3$	M1	Uses $\tan 2x = k$ and works with "$2x$".
$2x = 180 - 71.6$ or $360 - 71.6$	M1	Finds "$2x$" before ÷ 2
$x = 54.2°$ or $144.2°$	A1A1√	co. co√ (both of these need 2nd M)
Also $234.2°$ and $324.2°$	A1√ [5]	for 180° + his answer(s)
(ii) 12 answers.	B1√ [1]	for 3 times the number of solns to (i).

Q8.

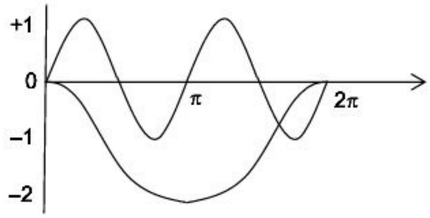
5 (i)	$\dfrac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$	M1	
	$\dfrac{\sin^2\theta-\sin\theta\cos\theta+\cos\theta\sin\theta-\cos^2\theta}{\sin^2\theta-\cos^2\theta}$	A1	
	$\dfrac{1}{\sin^2\theta-\cos^2\theta}$ **AG**	A1 [3]	www

(ii) $s^2-(1-s^2)=\frac{1}{3}$ or $1-c^2-c^2=\frac{1}{3}$ or $3(s^2-c^2)=c^2+s^2$	M1	Applying $c^2+s^2=1$
$\sin\theta=(\pm)\sqrt{\frac{2}{3}}$ or $\cos\theta=(\pm)\sqrt{\frac{1}{3}}$ or $\tan\theta=(\pm)\sqrt{2}$	A1	Or $s=(\pm)\ 0.816$, $c=(\pm)\ 0.577$, $t=(\pm)\ 1.414$
$\theta=54.7°, 125.3°, 234.7°, 305.3°$	A1A1 [4]	any 2 solutions for 1st A1 >4 solutions in range max A1A0

Q9.

3	$2\cos^2\theta=\tan^2\theta$			
(i)	$\rightarrow 2\cos^2\theta=\dfrac{\sin^2\theta}{\cos^2\theta}$	M1		Use of $t^2=s^2\div c^2$ or alternative. Correct eqn.
	$\rightarrow$ Uses $c^2+s^2=1 \rightarrow 2c^4=1-c^2$	A1	[2]	
(ii)	$(2c^2-1)(c^2+1)=0 \rightarrow c=\pm\dfrac{1}{\sqrt{2}}$	M1		Method of solving for 3-term quadratic.
	$\rightarrow \theta=\frac{1}{4}\pi$ or $\frac{3}{4}\pi$.	A1 A1√	[3]	(in terms of π). √ for $\pi-1^{st}$ ans. Cannot gain A1√ if other answers given in the range.

Q10.

5 **(i)**		B1 DB1 B1 DB1	[4]	y =sin2x has 2 cycles, starts and finishes on the *x*-axis, max comes first. From +1 to −1. Smooth curves. y = cosx − 1 has one cycle, starts and finishes on *x*-axis, with a minimum pt. From 0 to −2, smooth curve, flattens.
(ii)	**(a)** $\sin 2x = -\frac{1}{2}$ → 4 solutions	B1√	[1]	√ for their curve.
	(b) $\sin 2x + \cos x + 1 = 0$ → 3 solutions.	B1√	[1]	√ for intersections of their curves.

Q11.

1 $3\tan(2x + 15°) = 4$ $\tan(2x + 15°) = 1\frac{1}{3}$ Sets the bracket to $\tan^{-1}(1\frac{1}{3})$ $2x + 15 = 53.13°$ or $233.13°$ → $x = 19.1°$ or $109.1°$	M1 M1 A1 A1√ [4]	Removes the "3" first by division. Looks up $\tan^{-1}1\frac{1}{3}$, then uses bracket co. √ for (90 + 1[st] answer) and no other answers in the range.

Q12.

2	B1 B1 B1 B1 [4]	1 complete oscillation 0 → π Range from −3 to 3 All correct (V shape B0) Line correct.

Q13.

4	(i) $\frac{\sin x \tan x}{1-\cos x} = \frac{\sin^2 x}{\cos x(1-\cos x)}$	M1		Use of $\tan x = \sin x \div \cos x$
	$= \frac{1-\cos^2 x}{\cos x(1-\cos x)}$	M1		Use of $\sin^2 x = 1 - \cos^2 x$
	$= \frac{(1-\cos x)(1+\cos x)}{\cos x(1-\cos x)} = \frac{1}{\cos x} + 1$	M1	[3]	Realising the need to use difference of 2 squares. Answer given.
	(ii) $\frac{1}{\cos x} + 1 + 2 - 0$ $\rightarrow \cos x = -⅓$ $\rightarrow x = 109.5°$ or $250.5°$	M1 A1 A1√	[3]	Uses part **(i)** with cos x as subject. co. √ for 360° – 1st answer.

Q14.

7	$x \mapsto 3 - 2\tan(\frac{1}{2}x)$			
	(i) Range of f < 3	B1	[1]	co. Allow <
	(ii) $f(\frac{2}{3}\pi) = 3 - 2\sqrt{3}$	B1	[1]	co
	(iii)	B2, 1, 0 Indep.	[2]	Starting at $y = 3$ Shape correct – no turning points. Tending tangentially towards $x = \pi$
	(iv) $y = 3 - 2\tan\left(\frac{x}{2}\right)$	M1 M1		Attempt at making x the subject. Order of operations all ok.
	$\rightarrow f^{-1}(x) - 2\tan^{-1}\left(\frac{3-x}{2}\right)$	A1	[3]	co – but with x, not y.

Q15.

3	$15\cos^2 x + \cos x - 2 = 0$ $(5\cos x + 2)(3\cos x - 1) = 0$	M1 M1		$1 - \cos^2 x = \sin^2 x$ & attempt simplify Attempt to solve 3-term quadratic for cos$x$
	113(.6), 70.5	A1A1	[4]	SC 1.98, 1.23 scores 1/2

Q16.

4	(i) Correct sine curve	B1	[1]	2 shown or implied
	(ii) Required line $y = 1 - \frac{x}{\pi}$	B1		
	Line through $(0, 1)$, $(\pi, 0)$ drawn	B1		SC B1 for correct graphs without 1 or 2 marked
	3 roots	B1√	[3]	ft on trig curve and line

Q17.

3	(i) Correct cosine curve for at least 1 oscillation	B1		Range $-1 \to 1$. Ignore labels on θ axis
	Exactly 2 complete oscillations in $[0, 2\pi]$	B1		
	Line $y = \frac{1}{2}$ correct	B1	[3]	
	(ii) 4	B1√	[1]	Ft *their* graph. Accept 30°, 150°, 210°, 330°
	(iii) 20	B1√	[1]	Or $5 \times$ *their* part **(ii)**

Q18.

5	**(i)** $3\cos^2 x + 8\cos x + 4 = 0$ $(3\cos x + 2)(\cos x + 2) = 0$ $\cos x = -\frac{2}{3}$	M1 M1 A1	[3]	Use of $c^2 + s^2 = 1$ Factorising, formula or completing the square needed **AG** Ignore $\cos x = -2$ also offered SC B1 if –2/3 and –2 seen
	(ii) $\cos(\theta + 70) = -\frac{2}{3}$, $\theta = 61.8$ $\theta + 70 = 131.8$ (or 228.2) $\theta = 158.2$	M1 A1 M1 A1	[4]	

Q19.

7	**(i)** $2(1 - \sin^2\theta) = 3\sin\theta$ $(2\sin\theta - 1)(\sin\theta + 2) = 0$ $\theta = 30°$ or $150°$	**M1** **M1** **A1A1**	[4]	Use $c^2 + s^2 = 1$ Attempt to solve cao
	(ii) $n = \frac{\text{their } 30}{10} = 3$ $(\text{their } 3)\theta = 720 + \text{their}150 = 870$ $\theta = 290°$	**B1√** **M1** **A1**	[3]	ft provided *n* is an integer Allow full list up to at least 870 cao

Q20.

3	$7\cos x + 5 = 2(1 - \cos^2 x)$ $(2\cos x + 1)(\cos x + 3) = 0$ $\cos x = -0.5$ $x = 120°, 240°$	M1 A1 A1 A1√	[4]	Use of $c^2 + s^2 = 1$ ft for 360 –1st solution

Q21.

4 **(i)** $4(1-\cos^2 x)+8\cos x-7-0$	**M1**	Use c^2+s^2-1
$4c^2-8c+3-0 \rightarrow (2\cos x-1)(2\cos x-3)-0$	**M1**	Attempt to solve
$x-60°$ or $300°$	**A1A1** **[4]**	
(ii) $\frac{1}{2}\theta-60°$ (or $300°$)	**M1**	Allow $300°$ in addition
$\theta-120°$ only	**A1** **[2]**	

Q22.

7 **(a)** $x^2-1=\sin\frac{\pi}{3}$	M1	
$x-\pm 1.366$	A1A1√ [3]	√ for negative of 1st answer
(b) $2\theta+\frac{\pi}{3}-\frac{5\pi}{6}\left(\text{or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6}\right)$	B1	1 correct angle on RHS is sufficient
$2\theta-\frac{\pi}{2}-\left(\text{or } \frac{11\pi}{6}\right)$	M1	Isolating 2θ
$\theta-\frac{\pi}{4}, \frac{11\pi}{12}$	A1A1 [4]	SC decimals 0.785 & 2.88 scores M1B1