Q1.

7	(i)	Obtain modulus $\sqrt{8}$	B1	
		Obtain argument $\frac{1}{4}\pi$ or 45°	B1	[2]
	(ii)	Show 1, i and u in relatively correct positions on an Argand diagram	B1	
		Show the perpendicular bisector of the line joining 1 and i	B1	
		Show a circle with centre u and radius 1	B1	
		Shade the correct region	B1	[4]
	(iii)	State or imply relevance of the appropriate tangent from O to the circle	B1 √	
		Carry out complete strategy for finding $\lvert z \rvert$ for the critical point	M1	
		Obtain answer $\sqrt{7}$	A1	[3]

Q2.

8	(a)	*EITHER*:	Substitute $1+i\sqrt{3}$, attempt complete expansions of the x^3 and x^2 terms	M1	
			Use $i^2=-1$ correctly at least once	B1	
			Complete the verification correctly	A1	
			State that the other root is $1-i\sqrt{3}$	B1	
		*OR*1:	State that the other root is $1-i\sqrt{3}$	B1	
			State quadratic factor x^2-2x+4	B1	
			Divide cubic by 3-term quadratic reaching partial quotient $2x+k$	M1	
			Complete the division obtaining zero remainder	A1	
		*OR*2:	State factorisation $(2x+3)(x^2-2x+4)$, or equivalent	B1	
			Make reasonable solution attempt at a 3-term quadratic and use $i^2=-1$	M1	
			Obtain the root $1+i\sqrt{3}$	A1	
			State that the other root is $1-i\sqrt{3}$	B1	[4]
	(b)		Show point representing $1+i\sqrt{3}$ in relatively correct position on an Argand diagram	B1	
			Show circle with centre at $1+i\sqrt{3}$ and radius 1	B1√	
			Show line for $\arg z=\frac{1}{3}\pi$ making $\frac{1}{3}\pi$ with the real axis	B1	
			Show line from origin passing through centre of circle, or the diameter which would contain the origin if produced	B1	
			Shade the relevant region	B1√	[5]

Q3.

8	(i)	Either:	Multiply numerator and denominator by $(1 - 2i)$, or equivalent	M1	
			Obtain $-3i$	A1	
			State modulus is 3	A1	
			Refer to u being on negative imaginary axis or equivalent and confirm argument as $-\frac{1}{2}\pi$	A1	
		Or:	Using correct processes, divide moduli of numerator and denominator	M1	
			Obtain 3	A1	
			Subtract argument of denominator from argument of numerator	M1	
			Obtain $-\tan^{-1}\frac{1}{2} - \tan^{-1}2$ or $-0.464 - 1.107$ and hence $-\frac{1}{2}\pi$ or -1.57	A1	[4]
	(ii)		Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction	B1	
			Use correct trigonometry to find required value	M1	
			Obtain $\frac{3}{2}\sqrt{2}$ or equivalent	A1	[3]
	(iii)		Show, or imply, locus is a circle with centre $(1 + i)u$ and radius 1	M1	
			Use correct method to find distance from origin to furthest point of circle	M1	
			Obtain $3\sqrt{2} + 1$ or equivalent	A1	[3]

Q4.

7	(i)	Use the quadratic formula, completing the square, or the substitution $z = x + iy$ to find a root and use $i^2 = -1$	M1	
		Obtain final answers $-\sqrt{3} \pm i$, or equivalent	A1	[2]
	(ii)	State that the modulus of both roots is 2	B1√	
		State that the argument of $-\sqrt{3} + i$ is 150° or $\frac{5}{6}\pi$ (2.62) radians	B1√	
		State that the argument of $-\sqrt{3} - i$ is $-150°$ (or 210°) or $-\frac{5}{6}\pi$ (-2.62) radians or $\frac{7}{6}\pi$ (3.67) radians	B1√	[3]
	(iii)	Carry out an attempt to find the sixth power of a root	M1	
		Verify that one of the roots satisfies $z^6 = -64$	A1	
		Verify that the other root satisfies the equation	A1	[3]

Q5.

4	(i)	Either	Expand $(1+2\text{i})^2$ to obtain $-3+4\text{i}$ or unsimplified equivalent	B1	
			Multiply numerator and denominator by $2-\text{i}$	M1	
			Obtain correct numerator $-2+11\text{i}$ or correct denominator 5	A1	
			Obtain $-\frac{2}{5}+\frac{11}{5}\text{i}$ or equivalent	A1	
		Or	Expand $(1+2\text{i})^2$ to obtain $-3+4\text{i}$ or unsimplified equivalent	B1	
			Obtain two equations in x and y and solve for x or y	M1	
			Obtain final answer $x=-\frac{2}{5}$	A1	
			Obtain final answer $y=\frac{11}{5}$	A1	[4]
	(ii)		Draw a circle	M1	
			Show centre at relatively correct position, following their u	A1√	
			Draw circle passing through the origin	A1	[3]

Q6.

10	(a)		EITHER: Eliminate u or w and obtain an equation in w or in u	M1		
			Obtain a quadratic in u or w, e.g. $u^2-4\text{i}u-5=0$ or $w^2+4\text{i}w-5=0$	A1		
			Solve a 3-term quadratic for u or for w	M1		
			OR1: Having squared the first equation, eliminate u or w and obtain an equation in w or u	M1		
			Obtain a 2-term quadratic in u or w, e.g. $u^2=-3+4\text{i}$	A1		
			Solve a 2-term quadratic for u or for w	M1		
			OR2: Using $u=a+\text{i}b$, $w=c+\text{i}d$, equate real and imaginary parts and obtain 4 equations in a, b, c and d	M1		
			Obtain 4 correct equations	A1		
			Solve for a and b, or for c and d	M1		
			Obtain answer $u=1+2\text{i}$, $w=1-2\text{i}$	A1		
			Obtain answer $u=-1+2\text{i}$, $w=-1-2\text{i}$ and no other	A1	[5]	
	(b)	(i)	Show point representing $2-2\text{i}$ in relatively correct position	B1		
			Show a circle with centre $2-2\text{i}$ and radius 2	B1√		
			Show line for $\arg z=-\frac{1}{4}\pi$	B1		
			Show line for $\text{Re}\, z=1$	B1		
			Shade the relevant region	B1	[5]	
		(ii)	State answer $2+\sqrt{2}$, or equivalent (accept 3.41)	B1	[1]	

Q7.

7	(a)	State or imply $3a+3bi+2i(a-bi)=17+8i$	B1	
		Consider real and imaginary parts to obtain two linear equations in a and b	M1*	
		Solve two simultaneous linear equations for a or b	M1 (dep*)	
		Obtain $7-2i$	A1	[4]
	(b)	Either Show or imply a triangle with side 2	B1	
		State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$	B1	
		State or imply argument is $\frac{1}{4}\pi$	B1	
		Use sine rule or equivalent to find r	M1	
		Obtain $6.69e^{\frac{1}{4}\pi i}$	A1	
		Or State $y=x$.	B1	
		State $y=\frac{1}{\sqrt{3}}x+2$ or $\frac{\sqrt{3}}{2}=\frac{x}{\sqrt{x^2+(y-2)^2}}$ or $\frac{1}{2}=\frac{y-2}{\sqrt{x^2+(y-2)^2}}$	B1	
		State or imply argument is $\frac{\pi}{4}$	B1	
		Solve for x or y.	M1	
		Obtain $6.69e^{\frac{1}{4}\pi i}$	A1	[5]

Q8.

7	(i)	Show that $a^2+b^2=(a+ib)(a-ib)$	B1	
		Show that $(a+ib-ki)^* = a-ib+ki$	B1	[2]
	(ii)	Square both sides and express the given equation in terms of z and z^*	M1	
		Obtain a correct equation in any form, e.g. $(z-10i)(z^*+10i)=4(z-4i)(z^*+4i)$	A1	
		Obtain the given equation	A1	
		Either express $\lvert z-2i\rvert=4$ in terms of z and z^* or reduce the given equation to the form $\lvert z-u\rvert=r$	M1	
		Obtain the given answer correctly	A1	[5]
	(iii)	State that the locus is a circle with centre $2i$ and radius 5	B1	[1]

Q9.

7 (i)	Substitute $x = -2 + \text{i}$ in the equation and attempt expansion of $(-2 + \text{i})^3$	M1	
	Use $\text{i}^2 = -1$ correctly at least once and solve for k	M1	
	Obtain $k = 20$	A1	[3]
(ii)	State that the other complex root is $-2 - \text{i}$	B1	[1]

(iii)	Obtain modulus $\sqrt{5}$	B1	
	Obtain argument 153.4° or 2.68 radians	B1	[2]
(iv)	Show point representing u in relatively correct position in an Argand diagram	B1	
	Show vertical line through $z = 1$	B1	
	Show the correct half-lines from u of gradient zero and 1	B1	
	Shade the relevant region	B1	[4]
	[SR: For parts **(i)** and **(ii)** allow the following alternative method:		
	State that the other complex root is $-2 - \text{i}$	B1	
	State quadratic factor $x^2 + 4x + 5$	B1	
	Divide cubic by 3-term quadratic, equate remainder to zero and solve for k, or, using 3-term quadratic, factorise cubic and obtain k	M1	
	Obtain $k = 20$	A1]	

Q10.

6	(i)	State modulus is 2		B1	
		State argument is $\frac{1}{6}\pi$, or 30°, or 0.524 radians		B1	[2]
	(ii)	(a) State answer $3\sqrt{3}+\mathrm{i}$		B1	
		(b) *EITHER*:	Multiply numerator and denominator by $\sqrt{3}-\mathrm{i}$, or equivalent	M1	
			Simplify denominator to 4 or numerator to $2\sqrt{3}+2\mathrm{i}$	A1	
			Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR 1:	Obtain two equations in x and y and solve for x or for y	M1	
			Obtain $x=\frac{1}{2}\sqrt{3}$ or $y=\frac{1}{2}$	A1	
			Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR 2:	Using the correct processes express $\mathrm{i}z^*/z$ in polar form	M1	
			Obtain $x=\frac{1}{2}\sqrt{3}$ or $y=\frac{1}{2}$	A1	
			Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	[4]
	(iii)	Plot A and B in relatively correct positions		B1	
		EITHER:	Use fact that angle $AOB = \arg(\mathrm{i}z^*) - \arg z$	M1	
			Obtain the given answer	A1	
		OR 1:	Obtain tan $A\hat{O}B$ from gradients of OA and OB and the correct $\tan(A-B)$ formula	M1	
			Obtain the given answer	A1	
		OR 2:	Obtain cos $A\hat{O}B$ by using correct cosine formula or scalar product	M1	
			Obtain the given answer	A1	[3]

Q11.

3	(i)	Attempt multiplication and use $\mathrm{i}^2=-1$	M1	
		Obtain 3 + 4i	A1	
		Obtain 5 for <u>modulus</u>	B1	[3]
	(ii)	Draw complete circle with centre corresponding to their w^2 …	B1√	
		… and radius corresponding to their $\lvert w^2\rvert$	B1√	
		Shade the correct region	cwo B1	[3]

Q12.

10	**(a)**	*EITHER*:	Square x + iy and equate real and imaginary parts to 1 and $-2\sqrt{6}$ respectively	M1*	
			Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$	A1	
			Eliminate one variable and find an equation in the other	M1(dep*)	
			Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent	A1	
			Obtain answers $\pm(\sqrt{3} - \mathrm{i}\sqrt{2})$	A1	[5]
		OR:	Denoting $1 - 2\sqrt{6}\mathrm{i}$ by $R\mathrm{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\mathrm{cis}(\frac{1}{2}\theta)$ and find values of R and either cos θ or sin θ or tan θ	M1*	
			Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + \mathrm{i}\sin\frac{1}{2}\theta)$, and $\cos\theta = \frac{1}{5}$ or $\sin\theta = -\frac{2\sqrt{6}}{5}$ or $\tan\theta = -2\sqrt{6}$	A1	
			Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$	M1(dep*)	
			Obtain $\cos\frac{1}{2}\theta = \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta = \pm\sqrt{\frac{2}{5}}$, or equivalent	A1	
			Obtain answers $\pm(\sqrt{3} - \mathrm{i}\sqrt{2})$, or equivalent	A1	
			[Condone omission of ± except in the final answers.]		
	(b)		Show point representing 3i on a sketch of an Argand diagram	B1	
			Show a circle with centre at the point representing 3i and radius 2	B1√	
			Shade the interior of the circle	B1√	
			Carry out a complete method for finding the greatest value of arg z	M1	
			Obtain answer 131.8° or 2.30 (or 2.3) radians	A1	[5]
			[The f.t. is on solutions where the centre is at the point representing –3i.]		

Q13.

6	(i)	Use correct method for finding modulus of their w^2 or w^3 or both	M1	
		Obtain $\vert w^2 \vert = 2$ and $\vert w^3 \vert = 2\sqrt{2}$ or equivalent	A1	
		Use correct method for finding argument of their w^2 or w^3 or both	M1	
		Obtain $\arg(w^2) = -\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ and $\arg(w^3) = \frac{1}{4}\pi$	A1ft	[4]
	(ii)	Obtain centre $-\frac{1}{2} - \frac{1}{2}\mathrm{i}$ (their w^2)	B1ft	
		Calculate the diameter or radius using $\vert w - w^2 \vert$ w21 or right-angled triangle or cosine rule or equivalent	M1	
		Obtain radius $\frac{1}{2}\sqrt{10}$ or equivalent	A1	
		Obtain $\vert z + \frac{1}{2} + \frac{1}{2}\mathrm{i} \vert = \frac{1}{2}\sqrt{10}$ or equivalent	A1ft	[4]

Q14.

9	(i)	*EITHER*	Substitute $x = 1 + \sqrt{2}\,\mathrm{i}$ and attempt the expansions of the x^2 and x^4 terms	M1	
			Use $\mathrm{i}^2 = -1$ correctly at least once	B1	
			Complete the verification	A1	
			State second root $1 - \sqrt{2}\,\mathrm{i}$	B1	
		OR 1	State second root $1 - \sqrt{2}\,\mathrm{i}$	B1	
			Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,\mathrm{i}$	M1	
			Obtain $x^2 - 2x + 3$, or equivalent	A1	
			Show that the division of p(x) by $x^2 - 2x + 3$ gives zero remainder and complete the verification	A1	
		OR 2	Substitute $x = 1 + \sqrt{2}\,\mathrm{i}$ and use correct method to express x^2 and x^4 in polar form	M1	
			Obtain x^2 and x^4 in any correct polar form (allow decimals here)	B1	
			Complete an exact verification	A1	
			State second root $1 - \sqrt{2}\,\mathrm{i}$, or its polar equivalent (allow decimals here)	B1	[4]
	(ii)		Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}\,\mathrm{i}$	M1*	
			Obtain $x^2 - 2x + 3$, or equivalent	A1	
			Attempt division of p(x) by $x^2 - 2x + 3$ reaching a partial quotient $x^2 + kx$, or equivalent	M1 (dep*)	
			Obtain quadratic factor $x^2 - 2x + 2$	A1	
			Find the zeros of the second quadratic factor, using $\mathrm{i}^2 = -1$	M1 (dep*)	
			Obtain roots $-1 + \mathrm{i}$ and $-1 - \mathrm{i}$	A1	[6]

[The second M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + (6/3)$ and an equation in A and/or B]
[If part **(i)** is attempted by the *OR 1* method, then an attempt at part **(ii)** which uses or quotes relevant working or results obtained in part **(i)** should be marked using the scheme for part **(ii)**]

Q15.

10	(a)		Expand and simplify as far as $\mathrm{i}w^2 = -8\mathrm{i}$ or equivalent	B1	
			Obtain first answer $\mathrm{i}\sqrt{8}$, or equivalent	B1	
			Obtain second answer $-\mathrm{i}\sqrt{8}$, or equivalent and no others	B1	[3]
	(b)	(i)	Draw circle with centre in first quadrant	M1	
			Draw correct circle with interior shaded or indicated	A1	[2]
		(ii)	Identify ends of diameter corresponding to line through origin and centre	M1	
			Obtain $p = 3.66$ and $q = 7.66$	A1	
			Show tangents from origin to circle	M1	
			Evaluate $\sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$	M1	
			Obtain $\alpha = \frac{1}{4}\pi - \sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 0.424	A1	
			Obtain $\beta = \frac{1}{4}\pi + \sin^{-1}\left(\frac{1}{4}\sqrt{2}\right)$ or equivalent and hence 1.15	A1	[6]

Q16.

8	(a)	*EITHER:* Solve for u or for v	M1	
		Obtain $u = \dfrac{2i-6}{1-2i}$ or $v = \dfrac{5}{1-2i}$, or equivalent	A1	
		Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent		
		Or: Set u or v equal to x + iy, obtain two equations by equating real and imaginary parts and solve for x or for y	M1	
		OR: Using a + ib and c +id for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d	M1	
		Obtain $b + 2d = 2$, $a + 2c = 0$, $a + d = 0$ and $-b + c = 3$, or equivalent	A1	
		Solve for one unknown	M1	
		Obtain final answer $u = -2 - 2i$, or equivalent	A1	
		Obtain final answer $v = 1 + 2i$, or equivalent	A1	**[5]**
	(b)	Show a circle with centre –i	B1	
		Show a circle with radius 1	B1	
		Show correct half line from 2 at an angle of $\frac{3}{4}\pi$ to the real axis	B1	
		Use a correct method for finding the least value of the modulus	M1	
		Obtain final answer $\dfrac{3}{\sqrt{2}} - 1$, or equivalent, e.g. 1.12 (allow 1.1)	A1	**[5]**

Q17.

9	(a)	Solve using formula, including simplification under square root sign	M1*	
		Obtain $\dfrac{-2 \pm 4i}{2(2-i)}$ or similarly simplified equivalents	A1	
		Multiply by $\dfrac{2+i}{2+i}$ or equivalent in at least one case	M1(d*M)	
		Obtain final answer $-\frac{4}{5} + \frac{3}{5}i$	A1	
		Obtain final answer –i	A1	[5]

(b)	Show w in first quadrant with modulus and argument relatively correct	B1	
	Show w^3 in second quadrant with modulus and argument relatively correct	B1	
	Show w* in fourth quadrant with modulus and argument relatively correct	B1	
	Use correct method for area of triangle	M1	
	Obtain 10 by calculation	A1	[5]

Q18.

5	(i)	Either	Multiply numerator and denominator by $\sqrt{3}+\mathrm{i}$ and use $\mathrm{i}^2=-1$	M1	
			Obtain correct numerator $18+18\sqrt{3}\mathrm{i}$ or correct denominator 4	B1	
			Obtain $\frac{9}{2}+\frac{9}{2}\sqrt{3}\mathrm{i}$ or $\left(18+18\sqrt{3}\mathrm{i}\right)/4$	A1	
			Obtain modulus or argument	M1	
			Obtain $9e^{\frac{1}{3}\pi \mathrm{i}}$	A1	[5]
		OR	Obtain modulus and argument of numerator or denominator, or both moduli or both arguments	M1	
			Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$ or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees)	B1	
			Obtain $18e^{\frac{1}{6}\pi \mathrm{i}} \div 2e^{-\frac{1}{6}\pi \mathrm{i}}$ or equivalent	A1	
			Divide moduli and subtract arguments	M1	
			Obtain $9e^{\frac{1}{3}\pi \mathrm{i}}$	A1	[5]
	(ii)		State $3e^{\frac{1}{6}\pi \mathrm{i}}$, following through their answer to part (i)	B1√	
			State $3e^{\frac{1}{6}\pi \mathrm{i} \pm \frac{1}{2}\pi \mathrm{i}}$, following through their answer to part (i)	B1√	
			Obtain $3e^{-\frac{5}{6}\pi \mathrm{i}}$	B1	[3]

Q19.

7	(a)	*EITHER*:	Multiply numerator and denominator by $1-4\mathrm{i}$, or equivalent, and use $\mathrm{i}^2=-1$	M1	
			Simplify numerator to $-17-17\mathrm{i}$, or denominator to 17	A1	
			Obtain final answer $-1-\mathrm{i}$	A1	
		OR:	Using $\mathrm{i}^2=-1$, obtain two equations in x and y, and solve for x or for y	M1	
			Obtain $x=-1$ or $y=-1$, or equivalent	A1	
			Obtain final answer $-1-\mathrm{i}$	A1	**3**
	(b)	(i)	Show a point representing $2+\mathrm{i}$ in relatively correct position	B1	
			Show a circle with centre $2+\mathrm{i}$ and radius 1	B1√	
			Show the perpendicular bisector of the line segment joining i and 2	B1	
			Shade the correct region	B1	**4**
		(ii)	State or imply that the angle between the tangents from the origin to the circle is required	M1	
			Obtain answer 0.927 radians (or 53.1°)	A1	**2**

Q20.

5 (i) Substitute $z = 1 + i$ and obtain $w = \dfrac{1+2i}{1+i}$ B1

EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent M1
Simplify numerator to $3 + i$ or denominator to 2 A1
Obtain final answer $\frac{3}{2}+\frac{1}{2}i$, or equivalent A1

OR: Obtain two equations in x and y, and solve for x or for y M1
Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent A1
Obtain final answer $\frac{3}{2}+\frac{1}{2}i$, or equivalent A1 **[4]**

(ii) *EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $iz^2 + z - i = 0$ B1
Solve a 3-term quadratic for z or substitute $z = x + iy$ and use a correct method to solve for x and y M1

OR: Substitute $w = x + iy$ and obtain two correct equations in x and y by equating real and imaginary parts B1
Solve for x and y M1

Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\, i}{2i}$ A1

Obtain final answer $-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$ A1 **[4]**

Q21.

5 (i) State or imply $iw = -3 + 5i$ B1

Carry out multiplication by $\dfrac{4-i}{4-i}$ M1

Obtain final answer $-\dfrac{7}{17}+\dfrac{23}{17}i$ or equivalent A1 [3]

(ii) Multiply w by z to obtain $17 + 17i$ B1

State $\arg w = \tan^{-1}\dfrac{3}{5}$ or $\arg z = \tan^{-1}\dfrac{1}{4}$ B1

State $\arg wz = \arg w + \arg z$ M1

Confirm given result $\tan^{-1}\dfrac{3}{5}+\tan^{-1}\dfrac{1}{4}=\dfrac{1}{4}\pi$ legitimately A1 [4]