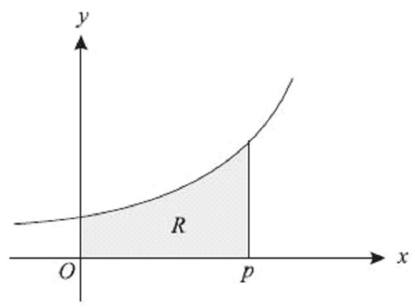
These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

3



The diagram shows the curve $y = e^{2x}$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = p$.

(i) Find, in terms of p, the area of R. [3]

(ii) Hence calculate the value of p for which the area of R is equal to 5. Give your answer correct to 2 significant figures. [3]

Q2.

7 (i) By expanding $\cos(2x + x)$, show that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x.$$ [5]

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{2}\pi} \cos^3 x \, dx = \tfrac{2}{3}.$$ [5]

Q3.

7 (i) By expanding $\sin(2x + x)$ and using double-angle formulae, show that

$$\sin 3x = 3\sin x - 4\sin^3 x.$$ [5]

(ii) Hence show that

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx = \tfrac{5}{24}.$$ [5]

Q4.

6 **(i)** Express $\cos^2 x$ in terms of $\cos 2x$. [1]

(ii) Hence show that

$$\int_0^{\frac{1}{3}\pi} \cos^2 x \, dx = \tfrac{1}{6}\pi + \tfrac{1}{8}\sqrt{3}.$$ [4]

(iii) By using an appropriate trigonometrical identity, deduce the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^2 x \, dx.$$

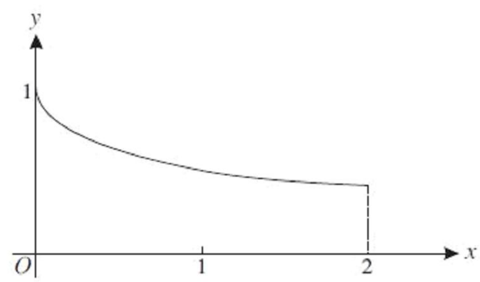
[3]

Q5.

3 Find the exact value of $\int_0^{\frac{1}{6}\pi} (\cos 2x + \sin x) \, dx$. [5]

Q6.

3



The diagram shows the curve $y = \frac{1}{1 + \sqrt{x}}$ for values of x from 0 to 2.

(i) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^2 \frac{1}{1 + \sqrt{x}} \, dx,$$

giving your answer correct to 2 decimal places. [3]

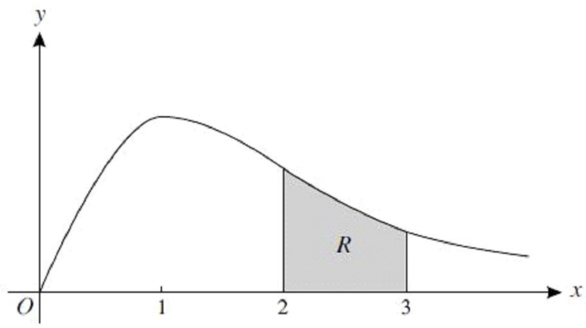
(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(i)**. [1]

Q7.

2 Show that $\int_0^6 \frac{1}{x+2}\,dx = 2\ln 2$. [4]

Q8.

2



The diagram shows part of the curve $y = xe^{-x}$. The shaded region R is bounded by the curve and by the lines $x = 2$, $x = 3$ and $y = 0$.

(i) Use the trapezium rule with two intervals to estimate the area of R, giving your answer correct to 2 decimal places. [3]

(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the area of R. [1]

Q9.

4 **(a)** Show that $\int_0^{\frac{1}{4}\pi} \cos 2x\,dx = \frac{1}{2}$. [2]

(b) By using an appropriate trigonometrical identity, find the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 3\tan^2 x\,dx.$$ [4]

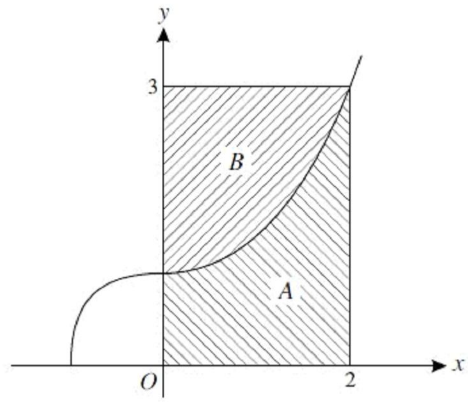
Q10.

6 **(a)** Find $\int 4e^x(3 + e^{2x})\,dx$. [4]

(b) Show that $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} (3 + 2\tan^2\theta)\,d\theta = \frac{1}{2}(8 + \pi)$. [4]

Q11.

2



The diagram shows the curve $y = \sqrt{(1 + x^3)}$. Region A is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. Region B is bounded by the curve and the lines $x = 0$ and $y = 3$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of region A. Give your answer correct to 2 decimal places. [3]

(ii) Deduce an approximation to the area of region B and explain why this approximation under-estimates the true area of region B. [2]

Q12.

4 **(a)** Find the value of $\int_0^{\overline{3}\pi} \sin\left(\frac{1}{2}x\right) \mathrm{d}x$. [3]

(b) Find $\int e^{-x}(1 + e^x) \, \mathrm{d}x$. [3]

Q13.

7 **(i)** Show that $(2\sin x + \cos x)^2$ can be written in the form $\frac{5}{2} + 2\sin 2x - \frac{3}{2}\cos 2x$. [5]

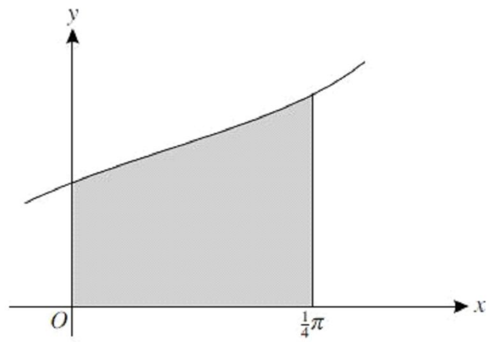
(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} (2\sin x + \cos x)^2 \, \mathrm{d}x$. [4]

Q14.

7 **(i)** Show that $\tan^2 x + \cos^2 x \equiv \sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}$ and hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} (\tan^2 x + \cos^2 x)\,dx.$$ [7]

(ii)



The region enclosed by the curve $y = \tan x + \cos x$ and the lines $x = 0$, $x = \frac{1}{4}\pi$ and $y = 0$ is shown in the diagram. Find the exact volume of the solid produced when this region is rotated completely about the x-axis. [4]

Q15.

3 **(i)** Show that $12\sin^2 x\cos^2 x \equiv \frac{3}{2}(1 - \cos 4x)$. [3]

(ii) Hence show that

$$\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} 12\sin^2 x\cos^2 x\,dx = \frac{\pi}{8} + \frac{3\sqrt{3}}{16}.$$ [3]

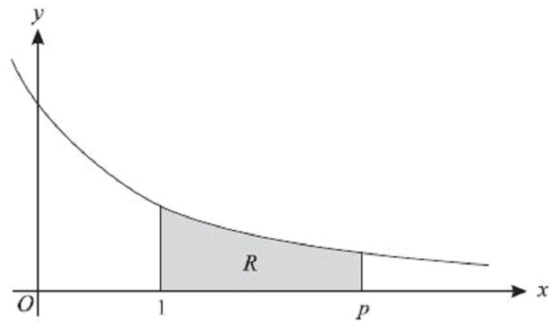
Q16.

1 A curve is such that $\frac{dy}{dx} = \frac{4}{7 - 2x}$. The point (3, 2) lies on the curve. Find the equation of the curve. [4]

Q17.

6 (a) Find the value of $\int_0^{\frac{1}{2}\pi} (\sin 2x + \cos x)\,dx$. [4]

(b)



The diagram shows part of the curve $y = \frac{1}{x+1}$. The shaded region R is bounded by the curve and by the lines $x = 1$, $y = 0$ and $x = p$.

(i) Find, in terms of p, the area of R. [3]

(ii) Hence find, correct to 1 decimal place, the value of p for which the area of R is equal to 2. [2]

Q18.

7 (i) Given that $y = \tan 2x$, find $\frac{dy}{dx}$. [2]

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{6}\pi} \sec^2 2x\,dx = \tfrac{1}{2}\sqrt{3},$$

and, by using an appropriate trigonometrical identity, find the exact value of $\int_0^{\frac{1}{6}\pi} \tan^2 2x\,dx$. [6]

(iii) Use the identity $\cos 4x \equiv 2\cos^2 2x - 1$ to find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{1}{1+\cos 4x}\,dx.$$ [2]

Q19.

1 Show that

$$\int_1^4 \frac{1}{2x+1}\,dx = \tfrac{1}{2}\ln 3.$$ [4]

Q20.

7 (i) Prove the identity

$$(\cos x + 3\sin x)^2 \equiv 5 - 4\cos 2x + 3\sin 2x. \qquad [4]$$

(ii) Using the identity, or otherwise, find the exact value of

$$\int_0^{\frac{1}{4}\pi} (\cos x + 3\sin x)^2 \, dx. \qquad [4]$$

Q21.

5 Show that $\int_1^2 \left(\frac{1}{x} - \frac{4}{2x+1}\right) dx = \ln \frac{18}{25}$. [6]

Q22.

5 (i) Express $\cos^2 2x$ in terms of $\cos 4x$. [2]

(ii) Hence find the exact value of $\int_0^{\frac{1}{8}\pi} \cos^2 2x \, dx$. [4]

Q23.

3 (i) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^{\frac{1}{3}\pi} \sec x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Using a sketch of the graph of $y = \sec x$ for $0 \leqslant x \leqslant \frac{1}{3}\pi$, explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i). [2]

Q24.

8 (a) Find the exact value of $\int_0^{\frac{1}{3}\pi} (\sin 2x + \sec^2 x) \, dx$. [5]

(b) Show that $\int_1^4 \left(\frac{1}{2x} + \frac{1}{x+1}\right) dx = \ln 5$. [4]

Q25.

3 Show that $\int_0^1 (e^x + 1)^2 \, dx = \frac{1}{2}e^2 + 2e - \frac{3}{2}$. [5]

Q26.

4 (a) Find $\int e^{1-2x} \, dx$. [2]

(b) Express $\sin^2 3x$ in terms of $\cos 6x$ and hence find $\int \sin^2 3x \, dx$. [4]

Q27.

2 Show that $\int_2^6 \frac{2}{4x+1} \, dx = \ln \frac{5}{3}$. [5]

Q28.

8 (i) By first expanding $\cos(2x + x)$, show that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x.$$ [5]

(ii) Hence show that

$$\int_0^{\frac{1}{6}\pi} (2\cos^3 x - \cos x) \, dx = \frac{5}{12}.$$ [5]

Q29.

4 Find the exact value of the positive constant k for which

$$\int_0^k e^{4x} \, dx = \int_0^{2k} e^x \, dx.$$ [6]

Q30.

4 (i) Express $\cos^2 x$ in terms of $\cos 2x$. [1]

(ii) Hence show that

$$\int_0^{\frac{1}{6}\pi} (\cos^2 x + \sin 2x) \, dx = \frac{1}{8}\sqrt{3} + \frac{1}{12}\pi + \frac{1}{4}.$$ [5]

Q31.

6 **(a)** Use the trapezium rule with two intervals to estimate the value of

$$\int_0^1 \frac{1}{6+2e^x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

(b) Find $\int \frac{(e^x-2)^2}{e^{2x}}\,dx$. [4]

Q32.

6 **(a)** Find $\int 4e^{-\frac{1}{2}x}\,dx$. [2]

(b) Show that $\int_1^3 \frac{6}{3x-1}\,dx = \ln 16$. [5]

Q33.

6 **(a)** Find

(i) $\int \frac{e^{2x}+6}{e^{2x}}\,dx$, [3]

(ii) $\int 3\cos^2 x\,dx$. [3]

(b) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_1^2 \frac{6}{\ln(x+2)}\,dx,$$

giving your answer correct to 2 decimal places. [3]

Q34.

1 **(i)** Find $\int \frac{2}{4x-1}\,dx$. [2]

(ii) Hence find $\int_1^7 \frac{2}{4x-1}\,dx$, expressing your answer in the form $\ln a$, where a is an integer. [3]

Q35.

6 **(a)** Find $\int (\sin x - \cos x)^2 \, dx$. [4]

(b) **(i)** Use the trapezium rule with 2 intervals to estimate the value of

$$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \operatorname{cosec} x \, dx,$$

giving your answer correct to 3 decimal places. [3]

(ii) Using a sketch of the graph of $y = \operatorname{cosec} x$ for $0 < x \leqslant \frac{1}{2}\pi$, explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(i)**. [2]

Q36.

5 **(i)** Prove that $\tan\theta + \cot\theta \equiv \dfrac{2}{\sin 2\theta}$. [3]

(ii) Hence

(a) find the exact value of $\tan\frac{1}{8}\pi + \cot\frac{1}{8}\pi$, [2]

(b) evaluate $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{6}{\tan\theta + \cot\theta} \, d\theta$. [3]

Q37.

6 **(a)** Show that $\displaystyle\int_6^{16} \frac{6}{2x - 7} \, dx = \ln 125$. [5]

(b) Use the trapezium rule with four intervals to find an approximation to

$$\int_1^{17} \log_{10} x \, dx,$$

giving your answer correct to 3 significant figures. [3]

Q38.

3 **(a)** Find $\int 4\cos\left(\frac{1}{3}x + 2\right) dx$. [2]

(b) Use the trapezium rule with three intervals to find an approximation to

$$\int_0^{12} \sqrt{(4 + x^2)} \, dx,$$

giving your answer correct to 3 significant figures. [3]

Q39.

1 Use the trapezium rule with four intervals to find an approximation to

$$\int_1^5 |2^x - 8| \, dx.$$ [3]

Q40.

3 (a) Find $\int 4\cos^2(\frac{1}{2}\theta)\, d\theta$. [3]

(b) Find the exact value of $\int_{-1}^{6} \frac{1}{2x+3}\, dx$. [4]

Q41.

2 (i) Find $\int_0^a (e^{-x} + 6e^{-3x})\, dx$, where a is a positive constant. [4]

(ii) Deduce the value of $\int_0^{\infty} (e^{-x} + 6e^{-3x})\, dx$. [1]

P3 (variant1 and 3)

Q1.

4 (i) Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\tfrac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x.$$ [3]

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x \, dx = \tfrac{1}{8}\sqrt{3}.$$ [3]

Q2.

8 (i) Express $\frac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part (i), show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}.$$ [2]

(iii) Hence show that $\int_0^1 \frac{4}{(x+1)^2(x+3)^2}\, dx = \frac{7}{12} - \ln\frac{3}{2}$. [5]

Q3.

7 (i) Prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [4]

(ii) Using this result, find the exact value of

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \cos^3\theta \, d\theta.$$ [4]

Q4.

7 The integral I is defined by $I = \int_0^2 4t^3 \ln(t^2+1)\, dt$.

(i) Use the substitution $x = t^2 + 1$ to show that $I = \int_1^5 (2x-2)\ln x \, dx$. [3]

(ii) Hence find the exact value of I. [5]

Q5.

10 The number of birds of a certain species in a forested region is recorded over several years. At time t years, the number of birds is N, where N is treated as a continuous variable. The variation in the number of birds is modelled by

$$\frac{dN}{dt} = \frac{N(1800-N)}{3600}.$$

It is given that $N = 300$ when $t = 0$.

(i) Find an expression for N in terms of t. [9]

(ii) According to the model, how many birds will there be after a long time? [1]

Q6.

3 Show that $\int_0^1 (1-x)e^{-\frac{1}{2}x}\, dx = 4e^{-\frac{1}{2}} - 2$. [5]

Q7.

9 In a chemical reaction, a compound X is formed from two compounds Y and Z. The masses in grams of X, Y and Z present at time t seconds after the start of the reaction are x, $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.01(10 - x)(20 - x).$$ [1]

(ii) Solve this differential equation and obtain an expression for x in terms of t. [9]

(iii) State what happens to the value of x when t becomes large. [1]

Q8.

7 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation and hence find the value of y when $x = 0.5$, giving your answer correct to 2 decimal places. [8]

Q9.

5 Given that $y = 0$ when $x = 1$, solve the differential equation

$$xy\frac{dy}{dx} = y^2 + 4,$$

obtaining an expression for y^2 in terms of x. [6]

Q10.

4 Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{x} - \frac{x}{4},$$

obtaining an expression for x^2 in terms of t. [7]

Q11.

9 By first expressing $\frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}\,dx = 8 - \ln 9.$$ [10]

Q12.

5 In a certain chemical process a substance A reacts with another substance B. The masses in grams of A and B present at time t seconds after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.6xy$ and $x = 5e^{-3t}$. When $t = 0$, $y = 70$.

(i) Form a differential equation in y and t. Solve this differential equation and obtain an expression for y in terms of t. [6]

(ii) The percentage of the initial mass of B remaining at time t is denoted by p. Find the exact value approached by p as t becomes large. [2]

Q13.

8 Let $f(x) = \frac{4x^2 - 7x - 1}{(x+1)(2x-3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_2^6 f(x)\,dx = 8 - \ln\left(\frac{49}{3}\right)$. [5]

Q14.

8 (a) Show that $\int_2^4 4x \ln x \, dx = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\int_0^{\frac{1}{24}\pi} \cos^3 4x \, dx$. [5]

Q15.

10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is $V\,\text{cm}^3$. The liquid is flowing into the tank at a constant rate of $80\,\text{cm}^3$ per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to $kV\,\text{cm}^3$ per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80e^{-kt}).$$ [7]

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation

$$k = \frac{4 - 4e^{-15k}}{25}.$$

Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

Q16.

4 **(i)** Express $(\sqrt{3})\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{\left((\sqrt{3})\cos x + \sin x\right)^2}\,dx = \tfrac{1}{4}\sqrt{3}.$$ [4]

Q17.

8 The variables x and t satisfy the differential equation

$$t\frac{dx}{dt} = \frac{k - x^3}{2x^2},$$

for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x as t becomes large. [1]

Q18.

5 (i) Prove the identity $\cos 4\theta - 4\cos 2\theta + 3 \equiv 8\sin^4\theta$. [4]

(ii) Using this result find, in simplified form, the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin^4\theta \, d\theta.$$ [4]

Q19.

10 In a model of the expansion of a sphere of radius r cm, it is assumed that, at time t seconds after the start, the rate of increase of the surface area of the sphere is proportional to its volume. When $t = 0$, $r = 5$ and $\frac{dr}{dt} = 2$.

(i) Show that r satisfies the differential equation

$$\frac{dr}{dt} = 0.08r^2.$$ [4]

[The surface area A and volume V of a sphere of radius r are given by the formulae $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$.]

(ii) Solve this differential equation, obtaining an expression for r in terms of t. [5]

(iii) Deduce from your answer to part (ii) the set of values that t can take, according to this model. [1]

Q20.

5 Let $I = \int_0^1 \frac{x^2}{\sqrt{(4 - x^2)}} \, dx$.

(i) Using the substitution $x = 2\sin\theta$, show that

$$I = \int_0^{\frac{1}{6}\pi} 4\sin^2\theta \, d\theta.$$ [3]

(ii) Hence find the exact value of I. [4]

Q21.

10 A certain substance is formed in a chemical reaction. The mass of substance formed t seconds after the start of the reaction is x grams. At any time the rate of formation of the substance is proportional to $(20 - x)$. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 1$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.05(20 - x). \qquad [2]$$

(ii) Find, in any form, the solution of this differential equation. [5]

(iii) Find x when $t = 10$, giving your answer correct to 1 decimal place. [2]

(iv) State what happens to the value of x as t becomes very large. [1]

Q22.

5 Show that $\int_0^7 \frac{2x + 7}{(2x + 1)(x + 2)}\,dx = \ln 50.$ [7]

Q23.

9 A biologist is investigating the spread of a weed in a particular region. At time t weeks after the start of the investigation, the area covered by the weed is A m^2. The biologist claims that the rate of increase of A is proportional to $\sqrt{(2A - 5)}$.

(i) Write down a differential equation representing the biologist's claim. [1]

(ii) At the start of the investigation, the area covered by the weed was 7 m^2 and, 10 weeks later, the area covered was 27 m^2. Assuming that the biologist's claim is correct, find the area covered 20 weeks after the start of the investigation. [9]

Q24.

4 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

Q25.

8 Let $f(x) = \dfrac{12 + 8x - x^2}{(2-x)(4+x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{2-x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\int_0^1 f(x)\,dx = \ln\left(\frac{25}{2}\right)$. [5]

Q26.

4 During an experiment, the number of organisms present at time t days is denoted by N, where N is treated as a continuous variable. It is given that

$$\frac{dN}{dt} = 1.2e^{-0.02t}N^{0.5}.$$

When $t = 0$, the number of organisms present is 100.

(i) Find an expression for N in terms of t. [6]

(ii) State what happens to the number of organisms present after a long time. [1]

Q27.

10 (i) Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

$$\int_0^{\frac{1}{4}\pi} (\tan^{n+2}x + \tan^n x)\,dx = \frac{1}{n+1}.$$ [4]

(ii) Hence find the exact value of

(a) $\int_0^{\frac{1}{4}\pi} (\sec^4 x - \sec^2 x)\,dx$, [3]

(b) $\int_0^{\frac{1}{4}\pi} (\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x)\,dx$. [3]

Q28.

6 The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

Q29.

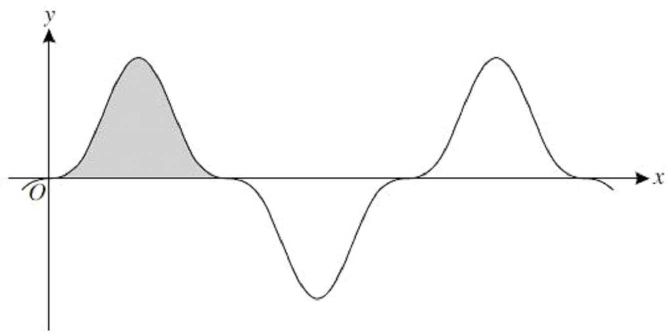
4 The variables x and y are related by the differential equation

$$(x^2+4)\frac{dy}{dx} = 6xy.$$

It is given that $y = 32$ when $x = 0$. Find an expression for y in terms of x. [6]

Q30.

7



The diagram shows part of the curve $y = \sin^3 2x \cos^3 2x$. The shaded region shown is bounded by the curve and the x-axis and its exact area is denoted by A.

(i) Use the substitution $u = \sin 2x$ in a suitable integral to find the value of A. [6]

(ii) Given that $\int_0^{k\pi} |\sin^3 2x \cos^3 2x| \, dx = 40A$, find the value of the constant k. [2]

Q31.

3 Find the exact value of $\int_1^4 \frac{\ln x}{\sqrt{x}} \, dx$. [5]

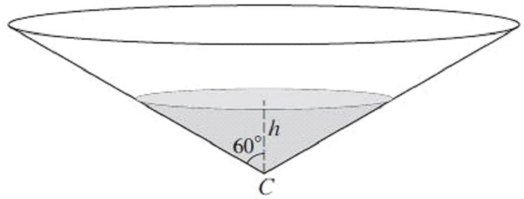
Q32.

5 **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta \, d\theta = \frac{1}{2}\ln 3$. [4]

Q33.

10



A tank containing water is in the form of a cone with vertex C. The axis is vertical and the semi-vertical angle is 60°, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is H. At this instant, a tap at C is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where h is the depth of water at time t. The tank becomes empty when $t = 60$.

(i) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where A is a positive constant. [4]

(ii) Solve the differential equation given in part **(i)** and obtain an expression for t in terms of h and H. [6]

(iii) Find the time at which the depth reaches $\frac{1}{2}H$. [1]

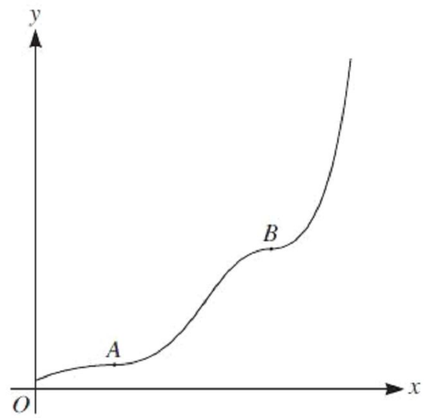
[The volume V of a cone of vertical height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.]

Q34.

2 Use the substitution $u = 3x + 1$ to find $\int \frac{3x}{3x+1}\,dx$. [4]

Q35.

10



A particular solution of the differential equation

$$3y^2 \frac{dy}{dx} = 4(y^3 + 1)\cos^2 x$$

is such that $y = 2$ when $x = 0$. The diagram shows a sketch of the graph of this solution for $0 \leqslant x \leqslant 2\pi$; the graph has stationary points at A and B. Find the y-coordinates of A and B, giving each coordinate correct to 1 decimal place. [10]

Q36.

2 Use the substitution $u = 1 + 3\tan x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3\tan x)}}{\cos^2 x} \, dx.$$ [5]

Q37.

4 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6ye^{3x}}{2 + e^{3x}}.$$

Given that $y = 36$ when $x = 0$, find an expression for y in terms of x. [6]

Q38.

5 The variables x and θ satisfy the differential equation

$$2\cos^2\theta \frac{dx}{d\theta} = \sqrt{(2x + 1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. [7]

Q39.

8 Let $\mathrm{f}(x) = \dfrac{6+6x}{(2-x)(2+x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2-x} + \dfrac{Bx+C}{2+x^2}$. [4]

(ii) Show that $\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = 3\ln 3$. [5]

Q40.

2 (i) Use the trapezium rule with 3 intervals to estimate the value of

$$\int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} \operatorname{cosec} x \,\mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

(ii) Using a sketch of the graph of $y = \operatorname{cosec} x$, explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part (i). [2]

Q41.

7 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{\mathrm{d}R}{\mathrm{d}x} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

Q42.

6 It is given that $I = \int_0^{0.3} \left(1+3x^2\right)^{-2} \mathrm{d}x$.

(i) Use the trapezium rule with 3 intervals to find an approximation to I, giving the answer correct to 3 decimal places. [3]

(ii) For small values of x, $\left(1+3x^2\right)^{-2} \approx 1 + ax^2 + bx^4$. Find the values of the constants a and b. Hence, by evaluating $\int_0^{0.3} (1 + ax^2 + bx^4)\,\mathrm{d}x$, find a second approximation to I, giving the answer correct to 3 decimal places. [5]

Q43.

8 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

Q44.

10 By first using the substitution $u = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\,dx = \ln\left(\tfrac{8}{5}\right).$$ [10]