Q1.

1	Use logarithms to linearise an equation	M1	
	Obtain $\frac{x}{y}=\frac{\ln 5}{\ln 2}$ or equivalent	A1	
	Obtain answer 2.32	A1	3

Q2.

2	(a)	Obtain a linear equation, e.g. $x \log 3 = \log 8$	B1	
		Obtain final answer 1.89	B1	2
	(b)	Use $2 \ln y = \ln(y^2)$	M1	
		Use law for addition or subtraction of logarithms	M1	
		Obtain answer $z=\frac{y+2}{y^2}$	A1	3

Q3.

2	(i)	State or imply $y \ln 3 = (x+2)\ln 4$	B1	
		State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	B1	
		State gradient is $\frac{\ln 4}{\ln 3}$, or equivalent (allow 1.26)	B1	[3]
	(ii)	Substitute $y = 2x$ and obtain a linear equation in x	M1*	
		Solve for x	M1(dep*)	
		Obtain answer 3.42	A1	[3]

Q4.

2	Use law for the logarithm of a product, a quotient or a power	M1*	
	Obtain $x\ln 4 = \ln 2 + x\ln 3$, or equivalent	A1	
	Solve for x	M1 (dep*)	
	Obtain answer $x = 2.41$	A1	[4]

Q5.

1	Use logarithms to linearise an equation	M1	
	Obtain $\frac{x}{y}=\frac{\ln 2.5}{\ln 1.25}$, or equivalent	A1	
	Obtain answer 4.11	A1√	[3]

Q6.

5	(i)	State or imply $2^{-x} = \frac{1}{y}$, or $2^{-x} = y^{-1}$	B1	
		Substitute and obtain a 3-term quadratic in y	M1	
		Obtain the given answer correctly	A1	[3]
	(ii)	Solve the given quadratic and carry out correct method for solving an equation of the form $2^x = a$, where $a > 0$	M1	
		Obtain answer $x = 1.58$ or 1.585	A1	
		Obtain answer $x = 0$	B1	[3]

Q7.

1	State or imply $y \log 2.8 = x \log 13$	B1	
	Rearrange into form $y = \frac{\log 13}{\log 2.8} x$ or equivalent	B1	
	Obtain answer $k = 2.49$	B1	[3]

Q8.

3	State or imply that $\ln y = \ln K + m \ln x$	B1	
	Equate intercept on axis for $\ln y$ to $\ln K$	M1	
	Obtain 7.39 for K	A1	
	Attempt calculation of gradient of line	M1	
	Obtain 1.37 for m	A1	[5]

Q9.

1	Attempt use of power law for logarithms	M1*	
	Obtain $x\log3 = x\log2 + 2\log2$ or equivalent	A1	
	Attempt solution for x of linear equation	M1	dep*
	Obtain 3.42	A1	[4]

Q10.

2	State or imply that $\ln y = \ln A + x \ln b$	B1	
	Equate intercept on y-axis to $\ln A$	M1	
	Obtain $\ln A = 2.14$ and hence $A = 8.5$	A1	
	Attempt gradient of line or equivalent (or use of correct substitution)	M1	
	Obtain $0.47 = \ln b$ or equivalent and hence $b = 1.6$	A1	[5]

Q11.

2	(i)	State or imply equation in the form $(5^x)^2 + 5^x - 12 = 0$	B1	
		Attempt solution of quadratic equation for 5^x	M1	
		Obtain $5^x = 3$ only	A1	[3]
	(ii)	Use logarithms to solve equation of the form $5^x = k$ where $k > 0$	M1	
		Obtain 0.683	A1	[2]

Q12.

2	Use $2 \ln x = \ln(x^2)$	M1	
	Use law for addition or subtraction of logarithms	M1	
	Obtain correct quadratic equation in x	A1	
	Make reasonable solution attempt at a 3-term quadratic (dependent on previous M marks)	DM1	
	State $x = \frac{3}{5}$ and no other solutions	A1	[5]

Q13.

4	(i)	State or imply $(y + 1) \log 5 = 3x \log 2$	M1	
		State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	A1	[2]
	(ii)	State gradient is $\frac{3 \ln 2}{\ln 5}$, or equivalent, e.g. $3\log_5 2$	B1	
		State $(0, -1)$	B1	[2]

Q14.

3	(i)	State or imply that $9^x = y^2$	B1	1
	(ii)	Carry out recognisable solution method for quadratic in y	M1	
		Obtain $y = \frac{1}{2}$ and $y = 3$ from $2y^2 - 7y + 3 = 0$	A1	
		Use log method to solve an equation of the form $3^x = k$	M1	
		Obtain answer $x = -\frac{\ln 2}{\ln 3}$, or exact equivalent { Y. ANY base }	A1	
		State exact answer $x = 1$ (no penalty if logs used)	B1	5

Q15.

2	State or imply at any stage $\ln y = \ln k - x \ln a$	B1	
	Equate estimate of $\ln y$- intercept to $\ln k$	M1	
	Obtain value for k in the range 9.97 ± 0.51	A1	
	Calculate gradient of the line of data points	M1	
	Obtain value for a in the range 2.12 ± 0.11	A1	
			[5]

Q16.

2	Use logarithms to obtain an equation in ln x		M1	
	Obtain $\ln x = \frac{\ln 11}{(3.9-3.2)}$, or equivalent		A1	
	Obtain answer $x = 31$ (accept 30.7, 30.74)		A1	3

Q17.

2	(i) State or imply that $4^x = y^2$ $(=2^{2x})$		B1	1
	(ii) Carry out recognizable solution method for a quadratic equation in y		M1	
	Obtain $y = 3$ and $y = \frac{1}{3}$ from $3y^2 - 10y + 3 = 0$		A1	
	Use logarithmic method to solve an equation of the form $2^x = k$, where $k > 0$		M1	
	State answer 1.58		A1	
	State answer -1.58	**(A1 √ if ± 1.59)**	A1	5

Q18.

3	State or imply $\ln y = \ln A - x \ln b$	B1	
	State $\ln A = 1.3$	B1	
	Obtain $A = 3.67$	B1	
	Form a numerical expression for the gradient of the line	M1	
	Obtain $b = 1.28$	A1	[5]

Q19.

2	Use $\ln x^2 = 2\ln x$	B1	
	Obtain $3 - x^2 = x^2$, or equivalent	B1	
	Solve for x	M1	
	Obtain answer $x = 1.22$, having rejected $x = -1.22$	A1	[4]

Q20.

2	State or imply $2 \ln x = \ln(x^2)$	B1	
	Use law for the logarithm of a quotient or a product	M1	
	Remove logarithms and obtain $yx^2 = y + 5$, or equivalent	A1	
	Obtain answer $y = \frac{5}{x^2 - 1}$	A1	[4]

Q21.

2	Use law for the logarithm of a product, a quotient or a power	M1*	
	Obtain $x \log 5 = (2x+1)\log 2$, or equivalent	A1	
	Solve for x, via correct manipulative technique(s)	M1(dep*)	
	Obtain answer $x = 3.11$. Allow $x \in [3.10, 3.11]$	A1	[4]

Q22.

5	State or imply $\ln y = \ln A + x \ln b$	B1	
	Form a numerical expression for the gradient of the line	M1	
	Obtain $b = 1.65$	A1	
	Use gradient and one point correctly to find $\ln A$	M1	
	Obtain $\ln A = 0.1$	A1	
	Obtain $A = 1.11$	A1	[6]

Q23.

4	Carry out recognizable solution method for quadratic in 3^x	M1	
	Obtain $3^x = 5$ and $3^x = 2$	A1	
	Use logarithmic method to solve an equation of the form $3^x = k$, where $k > 0$	M1	
	State answer 1.46	A1	
	State answer 0.631	A1	[5]

Q24.

2	Use law for the logarithm of a product, a quotient or a power	M1*	
	Obtain $(x+1)\log 4 = (2x-3)\log 5$, or equivalent	A1	
	Solve for x	M1(dep*)	
	Obtain answer $x = 3.39$	A1	[4]

Q25.

3	Use $2\ln(x+3) = \ln(x+3)^2$	M1	
	Use law for addition or subtraction of logarithms	M1	
	Obtain correct quadratic expression in x	A1	
	Make reasonable solution attempt at a 3-term quadratic	M1	
	State $x = 9$ and no other solutions (condone $x = -1$ not deleted)	A1	[5]

Q26.

2	Use law for the logarithm of a product, a quotient or a power	M1*	
	Obtain $x \log 5 = (2x-1) \log 3$ or equivalent	A1	
	Solve for x	M1(dep*)	
	Obtain answer $x = 1.87$	A1	[4]

Q27.

5	State or imply $\ln y = \ln A - x \ln b$	B1	
	Form a numerical expression for the gradient of the line	M1	
	Obtain $b = 1.82$	A1	
	Use gradient and one point correctly to find $\ln A$	M1	
	Obtain $\ln A = 3.5$	A1	
	Obtain $A = 33.12$	A1	[6]

Q28.

5	State or imply $\ln y = \ln K + px\ln 2$	B1	
	Obtain at least one of $1.87 = \ln K + 1.35p\ln 2$, $3.81 = \ln K + 3.35p\ln 2$, $p\ln 2 = \frac{3.81-1.87}{3.35-1.35}$		
	or equivalents	B1	
	Solve equation(s) to find one constant, dependent on previous B1	M1	
	Obtain $p = 1.40$	A1	
	Substitute to attempt value of K	DM1	
	Obtain $\ln K = 0.5605$ and hence $K = 1.75$	A1	[6]

Q29.

2	State or imply $\ln y = \ln a + x\ln b$	B1	
	Equate $\ln b$ to numerical gradient of line	M1	
	Obtain $b = 1.85$	A1	
	Substitute to find value of a	M1	
	Obtain $a = 3.45$	A1	[5]

Q30.

4	(a)	Use power law to produce $\ln(x-4)^2$	B1	
		Apply logarithm laws to produce equation without logarithms	M1	
		Obtain $(x-4)^2 = 2x$ or equivalent	A1	
		Solve 3-term quadratic equation	DM1	
		Obtain (finally) $x = 8$ only	A1	[5]
	(b)	Apply logarithms and use power law (once)	M1	
		Obtain $\frac{\ln 10^{10}}{\ln 1.4}$ or equivalent as part of inequality or equation	A1	
		Conclude with single integer 69	A1	[3]

P3 (variant1 and 3)

Q1.

3	(i)	*EITHER:*	State or imply $n \ln x + \ln y = \ln C$	B1	
			Substitute x- and y-values and solve for n	M1	
			Obtain $n = 1.50$	A1	
			Solve for C	M1	
			Obtain $C = 6.00$	A1	
		OR:	Obtain two correct equations by substituting x- and y-values in $x^n y = C$	B1	
			Solve for n	M1	
			Obtain $n = 1.50$	A1	
			Solve for C	M1	
			Obtain $C = 6.00$	A1	[5]
	(ii)		State that the graph of $\ln y$ against $\ln x$ has equation $n\ln x + \ln y = \ln C$ which is *linear* in $\ln y$ and $\ln x$, or has equation of the form $nX + Y = \ln C$, where $X = \ln x$ and $Y = \ln y$, and is thus a straight line	B1	[1]

Q2.

2	(i)	State or imply $3 \ln y = \ln A + 2x$ at any stage	B1	
		State gradient is $\frac{2}{3}$, or equivalent	B1	[2]
	(ii)	Substitute $x = 0$, $\ln y = 0.5$ and solve for A	M1	
		Obtain $A = 4.48$	A1	[2]

Q3.

1	Use law for the logarithm of a product, power or quotient	M1*	
	Obtain a correct linear equation, e.g. $(2x-1)\ln 5 = \ln 2 + x \ln 3$	A1	
	Solve a linear equation for x	M1(dep*)	
	Obtain answer $x = 1.09$	A1	[4]
	[SR: Reduce equation to the form $a^x = b$ M1*, obtain $\left(\frac{25}{3}\right)^x = 10$ Al, use correct method to calculate value of x M1(dep*), obtain answer 1.09 A1.]		

Q4.

2	Use law of the logarithm of a power and a product or quotient and remove logarithms	M1	
	Obtain a correct equation in any form, e.g. $\frac{2x+3}{x^2} = 3$	A1	
	Solve 3-term quadratic obtaining at least one root	M1	
	Obtain final answer 1.39 only	A1	[4]

Q5.

2	Use law for the logarithm of a product, quotient or power	M1	
	Use ln e = 1 or exp(1) = 3	M1	
	Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$	A1	
	Rearrange as $y = (ex^3 - 1)^{-1}$, or equivalent	A1	[4]

Q6.

2	*EITHER*:	Use laws of indices correctly and solve a linear equation for 3^x, or for 3^{-x}	M1	
		Obtain 3^x, or 3^{-x} in any correct form, e.g. $3^x = \frac{3^2}{(3^2-1)}$	A1	
		Use correct method for solving $3^{\pm x} = a$ for x, where $a > 0$	M1	
		Obtain answer $x = 0.107$	A1	
	OR:	State an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(3^{x_n}+9)}{\ln 3} - 2$	B1	
		Use the formula correctly at least once	M1	
		Obtain answer $x = 0.107$	A1	
		Show that the equation has no other root but 0.107	A1	[4]
		[For the solution 0.107 with no relevant working, award B1 and a further B1 if 0.107 is shown to be the only root.]		

Q7.

2	Use law for the logarithm of a power, a quotient, or a product correctly at least once	M1	
	Use ln e = 1 or e = exp(1)	M1	
	Obtain a correct equation free of logarithms, e.g. $1 + x^2 = ex^2$	A1	
	Solve and obtain answer $x = 0.763$ only	A1	[4]
	[For the solution $x = 0.763$ with no relevant working give B1, and a further B1 if 0.763 is shown to be the only root.]		
	[Treat the use of logarithms to base 10 with answer 0.333 only, as a misread.]		
	[SR: Allow iteration, giving B1 for an appropriate formula, e.g. $x_{n+1} = \exp((\ln(1 + x_n^2) - 1)/2)$, M1 for using it correctly once, A1 for 0.763, and A1 for showing the equation has no other root but 0.763.]		

Q8.

1	Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent	B1	
	Solve a 3-term quadratic for e^x or for u	M1	
	Obtain simplified solution $e^x = 3$ or $u = 3$	A1	
	Obtain final answer $x = 1.10$ and no other	A1	[4]

Q9.

2	*EITHER*	Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1}	M1	
		Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x =$	A1	
		Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a \neq 0$	M1	
		Obtain answer $x = 1.14$	A1	
	OR	Use an appropriate iterative formula, e.g. $x_{n+1} =$, correctly, at least once	M1	
		Obtain answer 1.14	A1	
		Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	
		Show there is no other root	A1	[4]
		[For the solution $x = 1.14$ with no relevant working give B1, and a further B1 if 1.14 is shown to be the only solution.]		

Q10.

1	State or imply $\ln e = 1$	B1	
	Apply at least one logarithm law for product or quotient correctly (or exponential equivalent)	M1	
	Obtain $x + 5 = ex$ or equivalent and hence $\frac{5}{e-1}$	A1	[3]

Q11.

2	*EITHER:*	State or imply non-modular equation $2^2(3^x - 1)^2 = (3^x)^2$, or pair of equations $2(3^x - 1) = \pm 3^x$	M1	
		Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$)	A1	
	OR:	Obtain $3^x = 2$ by solving an equation or by inspection	B1	
		Obtain $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection	B1	
	Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$		M1	
	Obtain final answers 0.631 and –0.369		A1	**[4]**

Q12.

1	Apply at least one logarithm property correctly	*M1	
	Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent **without logarithm** involved	A1	
	Rearrange to express x in terms of a	M1	d*M
	Obtain $\frac{16}{a-8}$ or equivalent	A1	[4]

Q13.

6	**(i)**	Use law for the logarithm for a product or quotient or exponentiation AND for a power	M1	
		Obtain $(4x-5)^2(x+1)=27$	B1	
		Obtain given equation correctly $16x^3-24x^2-15x-2=0$	A1	[3]
	(ii)	Obtain $x=2$ is root or $(x-2)$ is a factor, or likewise with $x=-\frac{1}{4}$	B1	
		Divide by $(x-2)$ to reach a quotient of the form $16x^2+kx$	M1	
		Obtain quotient $16x^2+8x+1$	A1	
		Obtain $(x-2)(4x+1)^2$ or $(x-2)$, $(4x+1)$, $(4x+1)$	A1	[4]
	(iii)	State $x=2$ only	A1	[1]

Q14.

1	Use law of the logarithm of a quotient or product or $2=\log_{10}100$	M1	
	Remove logarithms and obtain $x+9=100x$, or equivalent	A1	
	Obtain answer $x=\frac{1}{11}$	A1	**3**

Q15.

1	Use law of the logarithm of a power	M1	
	Obtain a correct linear equation in any form, e.g. $x=(x-2)\ln 3$	A1	
	Obtain answer $x=22.281$	A1	**[3]**