These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

5 (i) By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has exactly one root. [3]

(ii) Verify by calculation that the root lies between 1.0 and 1.4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

Q2.

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{5}\left(4x_n + \frac{306}{x_n^4}\right),$$

with initial value $x_1 = 3$, converges to α.

(i) Use this iterative formula to find α correct to 3 decimal places, showing the result of each iteration. [3]

(ii) State an equation satisfied by α, and hence show that the exact value of α is $\sqrt[5]{306}$. [2]

Q3.

3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{2}{x_n^3},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iteration to calculate α correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

(ii) State an equation which is satisfied by α and hence find the exact value of α. [2]

Q4.

6 (i) By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation $x = 9e^{-2x}$. [2]

(ii) Verify, by calculation, that this root lies between 1 and 2. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}(\ln 9 - \ln x_n)$$

converges, then it converges to the root of the equation given in part **(i)**. [2]

(iv) Use the iterative formula, with $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q5.

5 (i) By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1.0 and 1.2. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{1}{3-x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{1}{3-x_n}\right),$$

with initial value $x_1 = 1.1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q6.

8 The constant a, where $a > 1$, is such that $\int_1^a \left(x + \frac{1}{x}\right) dx = 6$.

(i) Find an equation satisfied by a, and show that it can be written in the form

$$a = \sqrt{(13 - 2\ln a)}.$$ [5]

(ii) Verify, by calculation, that the equation $a = \sqrt{(13 - 2\ln a)}$ has a root between 3 and 3.5. [2]

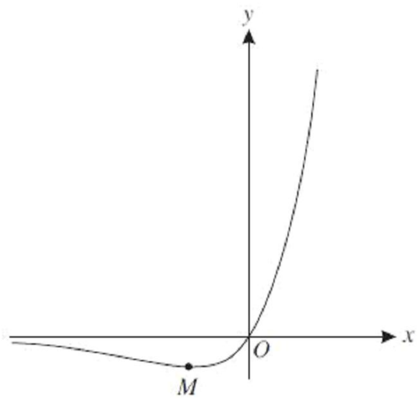
(iii) Use the iterative formula

$$a_{n+1} = \sqrt{(13 - 2\ln a_n)},$$

with $a_1 = 3.2$, to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q7.

7



The diagram shows the curve $y = xe^{2x}$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

(ii) Show that the curve intersects the line $y = 20$ at the point whose x-coordinate is the root of the equation

$$x = \frac{1}{2}\ln\left(\frac{20}{x}\right).$$ [1]

(iii) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{20}{x_n}\right),$$

with initial value $x_1 = 1.3$, to calculate the root correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

Q8.

7 (i) By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 2 - x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 0$ and $x = 0.5$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln(2 - x_n)$$

converges, then it converges to the root of the equation in part (i). [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q9.

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 1.3$ and $x = 1.4$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

converges, then it converges to the root of the equation in part (**i**). [1]

(iv) Use the iterative formula $x_{n+1} = \sqrt{(2 - \ln x_n)}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q10.

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 14 - x^2$$

has exactly two real roots. [3]

(ii) Show by calculation that the positive root lies between 1.2 and 1.3. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{2}\ln(14 - x^2).$$ [1]

(iv) Use an iteration process based on the equation in part (**iii**), with a suitable starting value, to find the root correct to 2 decimal places. Give the result of each step of the process to 4 decimal places. [3]

Q11.

3 The sequence $x_1, x_2, x_3, \ldots$ defined by

$$x_1 = 1, \quad x_{n+1} = \tfrac{1}{2}\sqrt[3]{(x_n^2 + 6)}$$

converges to the value α.

(i) Find the value of α correct to 3 decimal places. Show your working, giving each calculated value of the sequence to 5 decimal places. [3]

(ii) Find, in the form $ax^3 + bx^2 + c = 0$, an equation of which α is a root. [2]

Q12.

6 A curve has parametric equations

$$x = \frac{1}{(2t+1)^2}, \quad y = \sqrt{(t+2)}.$$

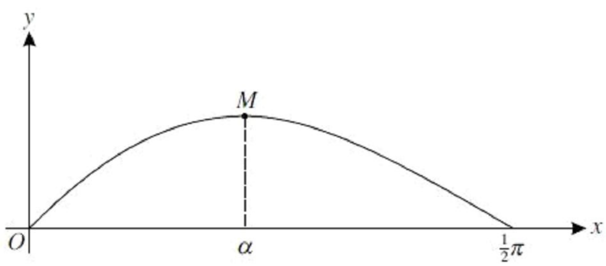
The point P on the curve has parameter p and it is given that the gradient of the curve at P is -1.

(i) Show that $p = (p+2)^{\frac{1}{6}} - \frac{1}{2}$. [6]

(ii) Use an iterative process based on the equation in part **(i)** to find the value of p correct to 3 decimal places. Use a starting value of 0.7 and show the result of each iteration to 5 decimal places. [3]

Q13.

6



The diagram shows the curve $y = \frac{\sin 2x}{x+2}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The x-coordinate of the maximum point M is denoted by α.

(i) Find $\frac{dy}{dx}$ and show that α satisfies the equation $\tan 2x = 2x + 4$. [4]

(ii) Show by calculation that α lies between 0.6 and 0.7. [2]

(iii) Use the iterative formula $x_{n+1} = \frac{1}{2}\tan^{-1}(2x_n + 4)$ to find the value of α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q14.

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 4x - 2,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.9$. [2]

(iii) Show that this root also satisfies the equation

$$x = \frac{1 + 2\tan x}{4\tan x}. \quad [1]$$

(iv) Use the iterative formula $x_{n+1} = \frac{1 + 2\tan x_n}{4\tan x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q15.

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$3e^x = 8 - 2x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.8$. [2]

(iii) Show that this root also satisfies the equation

$$x = \ln\left(\frac{8 - 2x}{3}\right). \quad [1]$$

(iv) Use the iterative formula $x_{n+1} = \ln\left(\frac{8 - 2x_n}{3}\right)$ to determine this root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q16.

4 (i) By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\sin x = \frac{1}{x^2}. \qquad [2]$$

(ii) Verify by calculation that this root lies between 1 and 1.5. [2]

(iii) Show that this value of x is also a root of the equation

$$x = \sqrt{(\operatorname{cosec} x)}. \qquad [1]$$

(iv) Use the iterative formula

$$x_{n+1} = \sqrt{(\operatorname{cosec} x_n)}$$

to determine this root correct to 3 significant figures, showing the value of each approximation that you calculate. [3]

Q17.

5 (i) By sketching a suitable pair of graphs, for $x < 0$, show that exactly one root of the equation $x^2 = 2^x$ is negative. [2]

(ii) Verify by calculation that this root lies between -1.0 and -0.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = -\sqrt{(2^{x_n})}$$

to determine this root correct to 2 significant figures, showing the result of each iteration. [3]

Q18.

6 (i) By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\cot x = x. \qquad [2]$$

(ii) Verify by calculation that this root lies between 0.8 and 0.9 radians. [2]

(iii) Show that this value of x is also a root of the equation

$$x = \tan^{-1}\left(\frac{1}{x}\right). \qquad [1]$$

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{x_n}\right)$$

to determine this root correct to 2 decimal places, showing the result of each iteration. [3]

Q19.

5 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation

$$\frac{1}{x} = \ln x.$$ [2]

(ii) Verify by calculation that this root lies between 1 and 2. [2]

(iii) Show that this root also satisfies the equation

$$x = e^{\frac{1}{x}}.$$ [1]

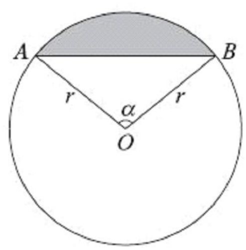
(iv) Use the iterative formula

$$x_{n+1} = e^{\frac{1}{x_n}},$$

with initial value $x_1 = 1.8$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q20.

5



The diagram shows a chord joining two points, A and B, on the circumference of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of the shaded segment is one sixth of the area of the circle.

(i) Show that α satisfies the equation

$$x = \tfrac{1}{3}\pi + \sin x.$$ [3]

(ii) Verify by calculation that α lies between $\tfrac{1}{2}\pi$ and $\tfrac{2}{3}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}\pi + \sin x_n,$$

with initial value $x_1 = 2$, to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q21.

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n}{3} + \frac{4}{x_n^2},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation that is satisfied by α and hence find the exact value of α. [2]

Q22.

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cos x = 2 - 2x,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

(iii) Show that, if a sequence of values given by the iterative formula

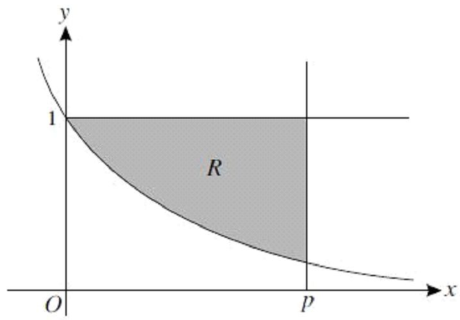
$$x_{n+1} = 1 - \tfrac{1}{2}\cos x_n$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.6$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q23.

7



The diagram shows the curve $y = e^{-x}$. The shaded region R is bounded by the curve and the lines $y = 1$ and $x = p$, where p is a constant.

(i) Find the area of R in terms of p. [4]

(ii) Show that if the area of R is equal to 1 then

$$p = 2 - e^{-p}.$$ [1]

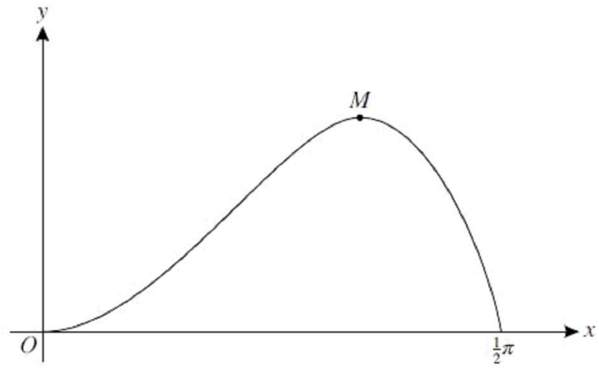
(iii) Use the iterative formula

$$p_{n+1} = 2 - e^{-p_n},$$

with initial value $p_1 = 2$, to calculate the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q24.

7



The diagram shows the curve $y = x^2 \cos x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Show by differentiation that the x-coordinate of M satisfies the equation

$$\tan x = \frac{2}{x}.$$ [4]

(ii) Verify by calculation that this equation has a root (in radians) between 1 and 1.2. [2]

(iii) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{2}{x_n}\right)$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q25.

6 The curve with equation $y = \frac{6}{x^2}$ intersects the line $y = x + 1$ at the point P.

(i) Verify by calculation that the x-coordinate of P lies between 1.4 and 1.6. [2]

(ii) Show that the x-coordinate of P satisfies the equation

$$x = \sqrt{\left(\frac{6}{x+1}\right)}.$$ [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt{\left(\frac{6}{x_n + 1}\right)},$$

with initial value $x_1 = 1.5$, to determine the x-coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q26.

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{7x_n}{8} + \frac{5}{2x_n^4},$$

with initial value $x_1 = 1.7$, converges to α.

(i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation that is satisfied by α and hence show that $\alpha = \sqrt[5]{20}$. [2]

Q27.

6 (i) Verify by calculation that the cubic equation

$$x^3 - 2x^2 + 5x - 3 = 0$$

has a root that lies between $x = 0.7$ and $x = 0.8$. [2]

(ii) Show that this root also satisfies an equation of the form

$$x = \frac{ax^2 + 3}{x^2 + b},$$

where the values of a and b are to be found. [2]

(iii) With these values of a and b, use the iterative formula

$$x_{n+1} = \frac{ax_n^2 + 3}{x_n^2 + b}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q28.

5 (i) By sketching a suitable pair of graphs, show that the equation

$$\frac{1}{x} = \sin x,$$

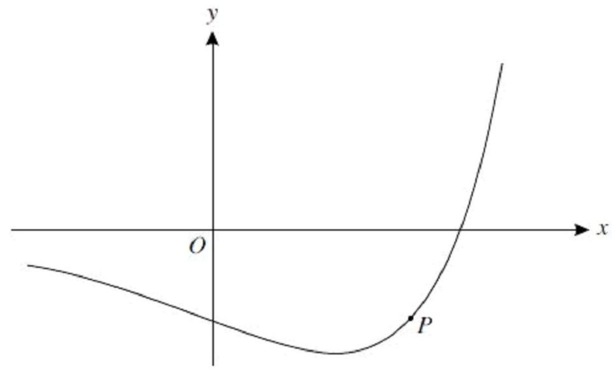
where x is in radians, has only one root for $0 < x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between $x = 1.1$ and $x = 1.2$. [2]

(iii) Use the iterative formula $x_{n+1} = \frac{1}{\sin x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q29.

7



The diagram shows the curve $y = (x-4)e^{\frac{1}{2}x}$. The curve has a gradient of 3 at the point P.

(i) Show that the x-coordinate of P satisfies the equation

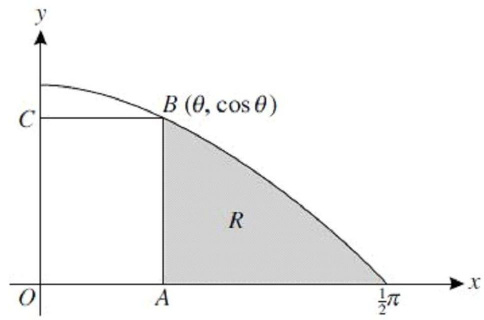
$$x = 2 + 6e^{-\frac{1}{2}x}.$$ [4]

(ii) Verify that the equation in part **(i)** has a root between $x = 3.1$ and $x = 3.3$. [2]

(iii) Use the iterative formula $x_{n+1} = 2 + 6e^{-\frac{1}{2}x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q30.

5



The diagram shows the curve $y = \cos x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. A rectangle $OABC$ is drawn, where B is the point on the curve with x-coordinate θ, and A and C are on the axes, as shown. The shaded region R is bounded by the curve and by the lines $x = \theta$ and $y = 0$.

(i) Find the area of R in terms of θ. [2]

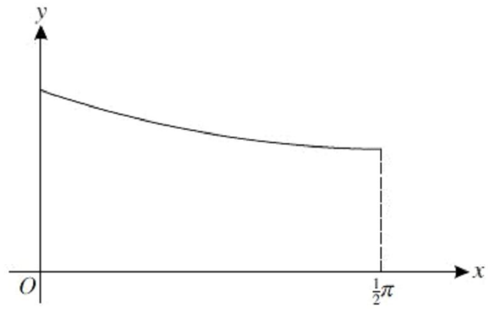
(ii) The area of the rectangle $OABC$ is equal to the area of R. Show that

$$\theta = \frac{1 - \sin\theta}{\cos\theta}.$$ [1]

(iii) Use the iterative formula $\theta_{n+1} = \dfrac{1 - \sin\theta_n}{\cos\theta_n}$, with initial value $\theta_1 = 0.5$, to determine the value of θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q31.

4



The diagram shows the part of the curve $y = \sqrt{(2 - \sin x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \sqrt{(2 - \sin x)}\, dx,$$

giving your answer correct to 2 decimal places. [3]

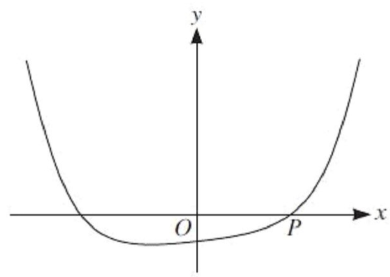
(ii) The line $y = x$ intersects the curve $y = \sqrt{(2 - \sin x)}$ at the point P. Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \sin x_n)}$$

to determine the x-coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q32.

2



The diagram shows the curve $y = x^4 + 2x - 9$. The curve cuts the positive x-axis at the point P.

(i) Verify by calculation that the x-coordinate of P lies between 1.5 and 1.6. [2]

(ii) Show that the x-coordinate of P satisfies the equation

$$x = \sqrt[3]{\left(\frac{9}{x} - 2\right)}.$$ [1]

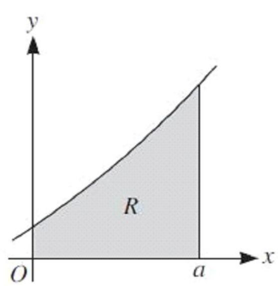
(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\left(\frac{9}{x_n} - 2\right)}$$

to determine the x-coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q33.

7



The diagram shows part of the curve $y = 8x + \frac{1}{2}e^x$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = a$, where a is positive. The area of R is equal to $\frac{1}{2}$.

(i) Find an equation satisfied by a, and show that the equation can be written in the form

$$a = \sqrt{\left(\frac{2 - e^a}{8}\right)}.$$ [5]

(ii) Verify by calculation that the equation $a = \sqrt{\left(\frac{2 - e^a}{8}\right)}$ has a root between 0.2 and 0.3. [2]

(iii) Use the iterative formula $a_{n+1} = \sqrt{\left(\frac{2 - e^{a_n}}{8}\right)}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q34.

4 **(i)** By sketching a suitable pair of graphs, show that the equation

$$3\ln x = 15 - x^3$$

has exactly one real root. [3]

(ii) Show by calculation that the root lies between 2.0 and 2.5. [2]

(iii) Use the iterative formula $x_{n+1} = \sqrt[3]{(15 - 3\ln x_n)}$ to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q35.

1 **(i)** Solve the equation $|x + 2| = |x - 13|$. [2]

(ii) Hence solve the equation $|3^y + 2| = |3^y - 13|$, giving your answer correct to 3 significant figures. [2]

Q36.

7 It is given that $\int_0^a \left(\frac{1}{2}e^{3x} + x^2\right) dx = 10$, where a is a positive constant.

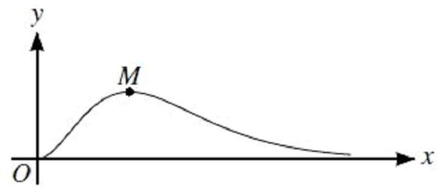
(i) Show that $a = \frac{1}{3}\ln(61 - 2a^3)$. [4]

(ii) Show by calculation that the value of a lies between 1.0 and 1.5. [2]

(iii) Use an iterative formula, based on the equation in part **(i)**, to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q37.

6



The diagram shows part of the curve $y = \dfrac{x^2}{1 + e^{3x}}$ and its maximum point M. The x-coordinate of M is denoted by m.

(i) Find $\dfrac{dy}{dx}$ and hence show that m satisfies the equation $x = \frac{2}{3}(1 + e^{-3x})$. [4]

(ii) Show by calculation that m lies between 0.7 and 0.8. [2]

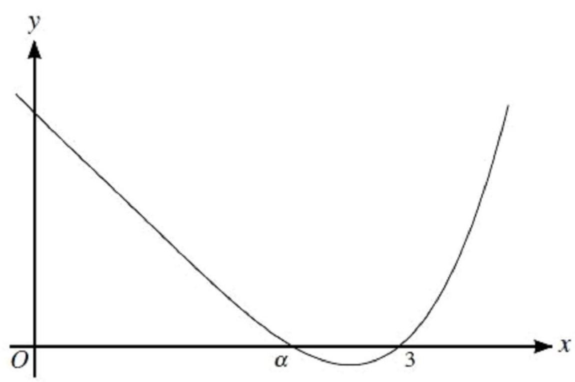
(iii) Use an iterative formula based on the equation in part **(i)** to find m correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q38.

1 Solve the equation $|3x - 1| = |2x + 5|$. [3]

Q39.

6



The polynomial p(x) is defined by

$$p(x) = x^4 - 3x^3 + 3x^2 - 25x + 48.$$

The diagram shows the curve $y = p(x)$ which crosses the x-axis at $(\alpha, 0)$ and $(3, 0)$.

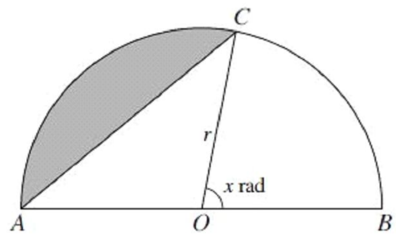
(i) Divide p(x) by a suitable linear factor and hence show that α is a root of the equation $x = \sqrt[3]{(16 - 3x)}$. [5]

(ii) Use the iterative formula $x_{n+1} = \sqrt[3]{(16 - 3x_n)}$ to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

P3 (variant1 and 3)

Q1.

6



The diagram shows a semicircle ACB with centre O and radius r. The angle BOC is x radians. The area of the shaded segment is a quarter of the area of the semicircle.

(i) Show that x satisfies the equation

$$x = \tfrac{3}{4}\pi - \sin x.$$ [3]

(ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{3}{4}\pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q2.

6 The curve $y = \dfrac{\ln x}{x+1}$ has one stationary point.

(i) Show that the x-coordinate of this point satisfies the equation

$$x = \frac{x+1}{\ln x},$$

and that this x-coordinate lies between 3 and 4. [5]

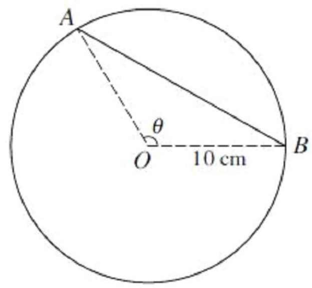
(ii) Use the iterative formula

$$x_{n+1} = \frac{x_n + 1}{\ln x_n}$$

to determine the x-coordinate correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q3.

6



The diagram shows a circle with centre O and radius 10 cm. The chord AB divides the circle into two regions whose areas are in the ratio 1 : 4 and it is required to find the length of AB. The angle AOB is θ radians.

(i) Show that $\theta = \frac{2}{5}\pi + \sin\theta$. [3]

(ii) Showing all your working, use an iterative formula, based on the equation in part **(i)**, with an initial value of 2.1, to find θ correct to 2 decimal places. Hence find the length of AB in centimetres correct to 1 decimal place. [5]

Q4.

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 0.8. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{1 + x_n^2}\right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q5.

1 Solve the equation $|4 - 2^x| = 10$, giving your answer correct to 3 significant figures. [3]

Q6.

10 (i) It is given that $2\tan 2x + 5\tan^2 x = 0$. Denoting $\tan x$ by t, form an equation in t and hence show that either $t = 0$ or $t = \sqrt[3]{(t + 0.8)}$. [4]

(ii) It is given that there is exactly one real value of t satisfying the equation $t = \sqrt[3]{(t + 0.8)}$. Verify by calculation that this value lies between 1.2 and 1.3. [2]

(iii) Use the iterative formula $t_{n+1} = \sqrt[3]{(t_n + 0.8)}$ to find the value of t correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

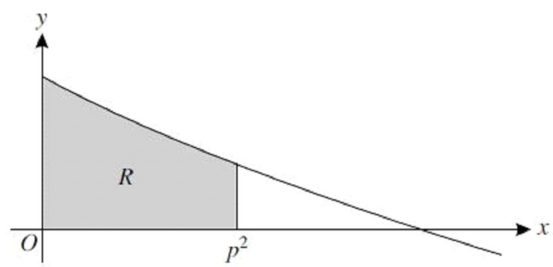
(iv) Using the values of t found in previous parts of the question, solve the equation

$$2\tan 2x + 5\tan^2 x = 0$$

for $-\pi \leqslant x \leqslant \pi$. [3]

Q7.

7



The diagram shows part of the curve $y = \cos(\sqrt{x})$ for $x \geqslant 0$, where x is in radians. The shaded region between the curve, the axes and the line $x = p^2$, where $p > 0$, is denoted by R. The area of R is equal to 1.

(i) Use the substitution $x = u^2$ to find $\int_0^{p^2} \cos(\sqrt{x})\,dx$. Hence show that $\sin p = \dfrac{3 - 2\cos p}{2p}$. [6]

(ii) Use the iterative formula $p_{n+1} = \sin^{-1}\left(\dfrac{3 - 2\cos p_n}{2p_n}\right)$, with initial value $p_1 = 1$, to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

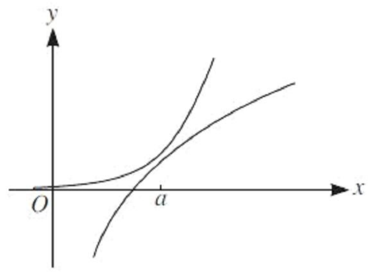
Q8.

4 (i) Solve the equation $|4x - 1| = |x - 3|$. [3]

(ii) Hence solve the equation $|4^{y+1} - 1| = |4^y - 3|$ correct to 3 significant figures. [3]

Q9.

6



The diagram shows the curves $y = e^{2x-3}$ and $y = 2\ln x$. When $x = a$ the tangents to the curves are parallel.

(i) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]

(ii) Verify by calculation that this equation has a root between 1 and 2. [2]

(iii) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

Q10.

3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{15}{x_n^3},$$

with initial value $x_1 = 3$, converges to α.

(i) Use this iterative formula to find α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

Q11.

4 (i) By sketching suitable graphs, show that the equation

$$4x^2 - 1 = \cot x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.6 and 1. [2]

(iii) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\sqrt{(1 + \cot x_n)}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q12.

7 (i) Given that $\int_1^a \frac{\ln x}{x^2}\,dx = \frac{2}{5}$, show that $a = \frac{5}{3}(1 + \ln a)$. [5]

(ii) Use an iteration formula based on the equation $a = \frac{5}{3}(1 + \ln a)$ to find the value of a correct to 2 decimal places. Use an initial value of 4 and give the result of each iteration to 4 decimal places. [3]

Q13.

5 (i) By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part (iii) to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q14.

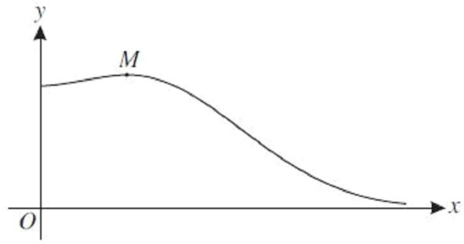
5 It is given that $\int_1^a x \ln x\,dx = 22$, where a is a constant greater than 1.

(i) Show that $a = \sqrt{\left(\frac{87}{2\ln a - 1}\right)}$. [5]

(ii) Use an iterative formula based on the equation in part (i) to find the value of a correct to 2 decimal places. Use an initial value of 6 and give the result of each iteration to 4 decimal places. [3]

Q15.

8



The diagram shows the curve $y = e^{-\frac{1}{2}x^2}\sqrt{(1 + 2x^2)}$ for $x \geqslant 0$, and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) The sequence of values given by the iterative formula

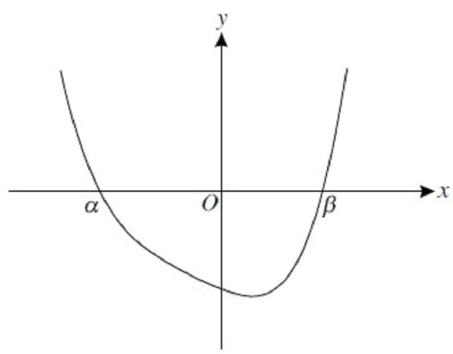
$$x_{n+1} = \sqrt{(\ln(4 + 8x_n^2))},$$

with initial value $x_1 = 2$, converges to a certain value α. State an equation satisfied by α and hence show that α is the x-coordinate of a point on the curve where $y = 0.5$. [3]

(iii) Use the iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q16.

6



The diagram shows the curve $y = x^4 + 2x^3 + 2x^2 - 4x - 16$, which crosses the x-axis at the points $(\alpha, 0)$ and $(\beta, 0)$ where $\alpha < \beta$. It is given that α is an integer.

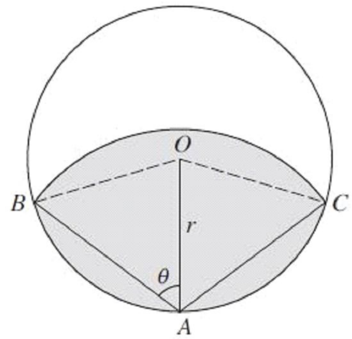
(i) Find the value of α. [2]

(ii) Show that β satisfies the equation $x = \sqrt[3]{(8 - 2x)}$. [3]

(iii) Use an iteration process based on the equation in part **(ii)** to find the value of β correct to 2 decimal places. Show the result of each iteration to 4 decimal places. [3]

Q17.

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is θ radians. The shaded region is bounded by the circumference of the circle and the arc with centre A joining B and C. The area of the shaded region is equal to half the area of the circle.

(i) Show that $\cos 2\theta = \dfrac{2\sin 2\theta - \pi}{4\theta}$. [5]

(ii) Use the iterative formula

$$\theta_{n+1} = \tfrac{1}{2}\cos^{-1}\left(\frac{2\sin 2\theta_n - \pi}{4\theta_n}\right),$$

with initial value $\theta_1 = 1$, to determine θ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

Q18.

5 It is given that $\int_0^p 4x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = 9$, where p is a positive constant.

(i) Show that $p = 2\ln\left(\dfrac{8p+16}{7}\right)$. [5]

(ii) Use an iterative process based on the equation in part **(i)** to find the value of p correct to 3 significant figures. Use a starting value of 3.5 and give the result of each iteration to 5 significant figures. [3]

Q19.

8 (i) By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation

$$\operatorname{cosec} x = x(\pi - x)$$

has exactly two real roots in the interval $0 < x < \pi$. [3]

(ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form $x = \dfrac{1 + x^2 \sin x}{\pi \sin x}$. [2]

(iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β, where $\alpha < \beta$.

(a) Use the iterative formula

$$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$

to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(b) Deduce the value of β correct to 2 decimal places. [1]

Q20.

4 The equation $x = \dfrac{10}{e^{2x} - 1}$ has one positive real root, denoted by α.

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2} \ln\left(1 + \frac{10}{x_n}\right)$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q21.

6 It is given that $\displaystyle\int_1^a \ln(2x)\,dx = 1$, where $a > 1$.

(i) Show that $a = \tfrac{1}{2} \exp\left(1 + \dfrac{\ln 2}{a}\right)$, where $\exp(x)$ denotes e^x. [6]

(ii) Use the iterative formula

$$a_{n+1} = \tfrac{1}{2} \exp\left(1 + \frac{\ln 2}{a_n}\right)$$

to determine the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q22.

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

(ii) Verify by calculation that the root lies between 3 and 4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(40 - \ln(x_n + 1))},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Deduce the root of the equation

$$(e^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]