

These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

2 The polynomial $x^4 - 9x^2 - 6x - 1$ is denoted by $f(x)$.

(i) Find the value of the constant a for which

$$f(x) \equiv (x^2 + ax + 1)(x^2 - ax - 1). \quad [3]$$

(ii) Hence solve the equation $f(x) = 0$, giving your answers in an exact form. [3]

Q2.

3 The cubic polynomial $2x^3 + ax^2 - 13x - 6$ is denoted by $f(x)$. It is given that $(x - 3)$ is a factor of $f(x)$.

(i) Find the value of a . [2]

(ii) When a has this value, solve the equation $f(x) = 0$. [4]

Q3.

4 The polynomial $x^3 - x^2 + ax + b$ is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, factorise $p(x)$. [2]

Q4.

4 The cubic polynomial $ax^3 + bx^2 - 3x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 1)$ and $(x + 2)$ are factors of $p(x)$.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, find the other linear factor of $p(x)$. [2]

Q5.

4 The polynomial $2x^3 - 3x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 2)$ the remainder is -20 .

(i) Find the values of a and b . [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 - 4)$. [3]

Q6.

- 4 The polynomial $2x^3 + 7x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5. Find the values of a and b . [5]

Q7.

- 6 The polynomial $x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 1)$ the remainder is 4.
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, find the other two linear factors of $p(x)$. [3]

Q8.

- 4 The polynomial $x^3 + 3x^2 + 4x + 2$ is denoted by $f(x)$.
- (i) Find the quotient and remainder when $f(x)$ is divided by $x^2 + x - 1$. [4]
- (ii) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. [2]

Q9.

- 7 The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(x - 3)$ the remainder is 30, and that when $p(x)$ is divided by $(x + 1)$ the remainder is 18.
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, verify that $(x - 2)$ is a factor of $p(x)$ and hence factorise $p(x)$ completely. [4]

Q10.

- 4 The polynomial $f(x)$ is defined by

$$f(x) = 3x^3 + ax^2 + ax + a,$$

where a is a constant.

- (i) Given that $(x + 2)$ is a factor of $f(x)$, find the value of a . [2]
- (ii) When a has the value found in part (i), find the quotient when $f(x)$ is divided by $(x + 2)$. [3]

Q11.

- 7 The cubic polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + ax^2 + bx + 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that, when $p(x)$ is divided by $(x + 1)$, the remainder is 24.

- (i) Find the values of a and b . [5]
(ii) When a and b have these values, factorise $p(x)$ completely. [3]

Q12.

- 3 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - 3x^2 - 5x + a + 4,$$

where a is a constant.

- (i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of a . [2]
(ii) When a has this value,
(a) factorise $p(x)$ completely, [3]
(b) find the remainder when $p(x)$ is divided by $(x + 1)$. [2]

Q13.

- 3 (i) Find the quotient when the polynomial

$$8x^3 - 4x^2 - 18x + 13$$

is divided by $4x^2 + 4x - 3$, and show that the remainder is 4. [3]

- (ii) Hence, or otherwise, factorise the polynomial

$$8x^3 - 4x^2 - 18x + 9. [2]$$

Q14.

- 4 The polynomial $ax^3 - 5x^2 + bx + 9$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 1)$ the remainder is 8.

- (i) Find the values of a and b . [5]
(ii) When a and b have these values, factorise $p(x)$ completely. [3]

Q15.

- 3 (i) The polynomial $2x^3 + ax^2 - ax - 12$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$. Find the value of a . [2]
- (ii) When a has this value, find the remainder when $p(x)$ is divided by $(x + 3)$. [2]

Q16.

- 2 The cubic polynomial $2x^3 + ax^2 + b$ is denoted by $f(x)$. It is given that $(x + 1)$ is a factor of $f(x)$, and that when $f(x)$ is divided by $(x + 2)$ the remainder is -5 . Find the values of a and b . [5]

Q17.

- 3 The polynomial $x^4 - 6x^2 + x + a$ is denoted by $f(x)$.
- (i) It is given that $(x + 1)$ is a factor of $f(x)$. Find the value of a . [2]
- (ii) When a has this value, verify that $(x - 2)$ is also a factor of $f(x)$ and hence factorise $f(x)$ completely. [4]

Q18.

- 4 The cubic polynomial $2x^3 - 5x^2 + ax + b$ is denoted by $f(x)$. It is given that $(x - 2)$ is a factor of $f(x)$, and that when $f(x)$ is divided by $(x + 1)$ the remainder is -6 . Find the values of a and b . [5]

Q19.

- 2 The polynomial $x^3 + 2x^2 + 2x + 3$ is denoted by $p(x)$.
- (i) Find the remainder when $p(x)$ is divided by $x - 1$. [2]
- (ii) Find the quotient and remainder when $p(x)$ is divided by $x^2 + x - 1$. [4]

Q20.

- 3 The polynomial $4x^3 - 7x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x - 3)$ is a factor of $p(x)$.
- (i) Show that $a = -3$. [2]
- (ii) Hence, or otherwise, solve the equation $p(x) = 0$. [4]

Q21.

- 5 The polynomial $3x^3 + 8x^2 + ax - 2$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.
- (i) Find the value of a . [2]
- (ii) When a has this value, solve the equation $p(x) = 0$. [4]

Q22.

- 2 The polynomial $2x^3 - x^2 + ax - 6$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.
- (i) Find the value of a . [2]
- (ii) When a has this value, factorise $p(x)$ completely. [3]

Q23.

- 3 The polynomial $4x^3 - 8x^2 + ax - 3$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$.
- (i) Find the value of a . [2]
- (ii) When a has this value, factorise $p(x)$ completely. [4]

Q24.

- 5 The polynomial $ax^3 + bx^2 - 5x + 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x - 2)$ are factors of $p(x)$.
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, find the other linear factor of $p(x)$. [2]

Q25.

- 7 The polynomial $3x^3 + 2x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 10.
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, solve the equation $p(x) = 0$. [4]

Q26.

- 3 The polynomial $x^3 + 4x^2 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that the remainder when $p(x)$ is divided by $(x + 1)$ is equal to the remainder when $p(x)$ is divided by $(x - 2)$.
- (i) Find the value of a . [3]
- (ii) When a has this value, show that $(x - 1)$ is a factor of $p(x)$ and find the quotient when $p(x)$ is divided by $(x - 1)$. [3]

Q27.

- 5 The polynomial $4x^3 + ax^2 + 9x + 9$, where a is a constant, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(2x - 1)$ the remainder is 10.
- (i) Find the value of a and hence verify that $(x - 3)$ is a factor of $p(x)$. [3]
- (ii) When a has this value, solve the equation $p(x) = 0$. [4]

Q28.

- 7 The polynomial $ax^3 - 3x^2 - 11x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, factorise $p(x)$ completely. [3]

Q29.

- 6 (i) The polynomial $x^4 + ax^3 - x^2 + bx + 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 1)$ and $(x + 2)$ are factors of $p(x)$. Find the values of a and b . [5]
- (ii) When a and b have these values, find the quotient when $p(x)$ is divided by $x^2 + x - 2$. [3]

Q30.

- 7 The polynomial $2x^3 - 4x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(x + 1)$ the remainder is 4, and that when $p(x)$ is divided by $(x - 3)$ the remainder is 12.
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, find the quotient and remainder when $p(x)$ is divided by $(x^2 - 2)$. [3]

Q31.

- 3 The polynomial $x^4 - 4x^3 + 3x^2 + 4x - 4$ is denoted by $p(x)$.
- (i) Find the quotient when $p(x)$ is divided by $x^2 - 3x + 2$. [3]
- (ii) Hence solve the equation $p(x) = 0$. [3]

Q32.

- 4 (i) The polynomial $x^3 + ax^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that when $p(x)$ is divided by $(x - 3)$ the remainder is 14, and that when $p(x)$ is divided by $(x + 2)$ the remainder is 24. Find the values of a and b . [5]
- (ii) When a and b have these values, find the quotient when $p(x)$ is divided by $x^2 + 2x - 8$ and hence solve the equation $p(x) = 0$. [4]

Q33.

- 4 (i) The polynomial $ax^3 + bx^2 - 25x - 6$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 3)$ and $(x + 2)$ are factors of $p(x)$. Find the values of a and b . [5]
- (ii) When a and b have these values, factorise $p(x)$ completely. [2]

Q34.

- 3 (i) Find the quotient when $6x^4 - x^3 - 26x^2 + 4x + 15$ is divided by $(x^2 - 4)$, and confirm that the remainder is 7. [3]
- (ii) Hence solve the equation $6x^4 - x^3 - 26x^2 + 4x + 8 = 0$. [3]

Q35.

- 6 The polynomial $p(x)$ is defined by

$$p(x) = x^3 + 2x + a,$$

where a is a constant.

- (i) Given that $(x + 2)$ is a factor of $p(x)$, find the value of a . [2]
- (ii) When a has this value, find the quotient when $p(x)$ is divided by $(x + 2)$ and hence show that the equation $p(x) = 0$ has exactly one real root. [5]

Q36.

- 5 (i) Given that $(x + 2)$ and $(x + 3)$ are factors of

$$5x^3 + ax^2 + b,$$

find the values of the constants a and b .

[4]

- (ii) When a and b have these values, factorise

$$5x^3 + ax^2 + b$$

completely, and hence solve the equation

$$5^{3y+1} + a \times 5^{2y} + b = 0,$$

giving any answers correct to 3 significant figures.

[5]

P3 (variant1 and 3)

Q1.

- 4 The polynomial $f(x)$ is defined by

$$f(x) = 12x^3 + 25x^2 - 4x - 12.$$

- (i) Show that $f(-2) = 0$ and factorise $f(x)$ completely.

[4]

- (ii) Given that

$$12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,$$

state the value of 3^y and hence find y correct to 3 significant figures.

[3]

Q2.

- 5 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

- (i) Find the values of a and b .

[5]

- (ii) When a and b have these values, find the quadratic factor of $p(x)$.

[2]

Q3.

3 The polynomial $p(x)$ is defined by

$$p(x) = x^3 - 3ax + 4a,$$

where a is a constant.

(i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of a . [2]

(ii) When a has this value,

(a) factorise $p(x)$ completely, [3]

(b) find all the roots of the equation $p(x^2) = 0$. [2]

Q4.

1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

Q5.

5 The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

Q6.

10 The polynomial $p(z)$ is defined by

$$p(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z + 2)$ is a factor of $p(z)$.

(i) Find the value of m . [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $p(z) = 0$, [5]

(b) the six roots of the equation $p(z^2) = 0$. [6]

Q7.

3 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a . [4]

(ii) When a has this value, find the real roots of the equation $p(x) = 0$. [2]

Q8.

- 7 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $p(x)$.

- (i) Find the value of a and hence factorise $p(x)$. [4]

- (ii) When a has the value found in part (i), express $\frac{8x - 13}{p(x)}$ in partial fractions. [5]

Q9.

- 3 The polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $f(x)$.

- (i) Find the value of a . [2]

- (ii) Show that, when a has this value, the equation $f(x) = 0$ has only one real root. [3]

Q10.

- 3 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b . [5]

Q11.

- 3 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

- (i) Find the values of a and b . [4]

- (ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

