Q1.

10	(i)	Express general point of l or m in component form, e.g. $(1 + s, 1 - s, 1 + 2s)$ or $(4 + 2t, 6 + 2t, 1 + t)$	B1	
		Equate at least two corresponding pairs of components and solve for s or t	M1	
		Obtain $s = -1$ or $t = -2$	A1	
		Verify that all three component equations are satisfied	A1	[4]
	(ii)	Carry out correct process for evaluating the scalar product of the direction vectors of l and m	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 74.2° (or 1.30 radians)	A1	[3]
	(iii)	*EITHER:* Use scalar product to obtain $a - b + 2c = 0$ and $2a + 2b + c = 0$	B1	
		Solve and obtain one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c = 5 : -3 : -4$, or equivalent	A1	
		Substitute coordinates of a relevant point and values for a, b and c in general equation of plane and evaluate d	M1	
		Obtain answer $5x - 3y - 4z = -2$, or equivalent	A1	
		OR 1: Using two points on l and one on m, or *vice versa*, state three equations in a, b, c and d	B1	
		Solve and obtain one ratio, e.g. $a : b$	M1	
		Obtain a ratio of three of the unknowns, e.g. $a : b : c = -5 : 3 : 4$	A1	
		Use coordinates of a relevant point and found ratio to find the fourth unknown, e.g. d	M1	
		Obtain answer $-5x + 3y + 4z = 2$, or equivalent	A1	
		OR 2: Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	B1	
		State three equations in x, y, z, λ and μ	M1	
		State three correct equations	A1	
		Eliminate λ and μ	M1	
		Obtain answer $5x - 3y - 4z = -2$, or equivalent	A1	
		OR 3: Attempt to calculate vector product of direction vectors of l and m	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$	A1	
		Form a plane equation and use coordinates of a relevant point to calculate d	M1	
		Obtain answer $-5x + 3y + 4z = 2$, or equivalent	A1	[5]

Q2.

10	**(i)**	Express general point of the line in component form, e.g. $(2+\lambda, -1+2\lambda, -4+2\lambda)$	B1	
		Substitute in plane equation and solve for λ	M1	
		Obtain position vector $4\mathbf{i}+3\mathbf{j}$, or equivalent	A1	[3]
	(ii)	State or imply a correct vector normal to the plane, e.g. $3\mathbf{i}-\mathbf{j}+2\mathbf{k}$	B1	
		Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine or inverse sine of the result	M1	
		Obtain answer 26.5° (or 0.462 radians)	A1	[4]
	(iii)	*EITHER*: State $a+2b+2c=0$ or $3a-b+2c=0$	B1	
		Obtain two relevant equations and solve for one ratio, e.g. $a:b$	M1	
		Obtain $a:b:c=6:4:-7$, or equivalent	A1	
		Substitute coordinates of a relevant point in $6x+4y-7z=d$ and evaluate d	M1	
		Obtain answer $6x+4y-7z=36$, or equivalent	A1	
		*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}+2\mathbf{j}+2\mathbf{k})\times(3\mathbf{i}-\mathbf{j}+2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $6\mathbf{i}+4\mathbf{j}-7\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $6x+4y-7z=d$ and evaluate d	M1	
		Obtain answer $6x+4y-7z=36$, or equivalent	A1	
		*OR*2: Attempt to form 2-parameter equation with relevant vectors	M1	
		State a correct equation, e.g. $\mathbf{r}=2\mathbf{i}-\mathbf{j}-4\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})+\mu(3\mathbf{i}-\mathbf{j}+2\mathbf{k})$	A1	
		State three equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain answer $6x+4y-7z=36$, or equivalent	A1	[5]

Q3.

3	**(i)**	Obtain $\pm\begin{pmatrix}3\\-4\\6\end{pmatrix}$ as normal to plane	B1	
		Form equation of p as $3x-4y+6z=k$ or $-3x+4y-6z=k$ and use relevant point to find k	M1	
		Obtain $3x-4y+6z=80$ or $-3x+4y-6z=-80$	A1	[3]
	(ii)	State the direction vector $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or equivalent	B1	
		Carry out correct process for finding scalar product of two relevant vectors	M1	
		Use correct complete process with moduli and scalar product and evaluate $\sin^{-1}$ or $\cos^{-1}$ of result	M1	
		Obtain 30.8° or 0.538 radians	A1	[4]

Q4.

10 **(i)**	*EITHER*:	Express general point of l or m in component form, e.g. $(2+\lambda, -\lambda, 1+2\lambda)$ or $(\mu, 2+2\mu, 6-2\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ (possible answers for λ are -2, $\frac{1}{4}$, 7 and for μ are 0, $2\frac{1}{4}$, $-4\frac{1}{2}$)	A1	
		Verify that all three component equations are not satisfied	A1	
	OR:	State a relevant scalar triple product, e.g. $(2\mathbf{i}-2\mathbf{j}-5\mathbf{k}).((\mathbf{i}-\mathbf{j}+2\mathbf{k})\times(\mathbf{i}+2\mathbf{j}-2\mathbf{k}))$	B1	
		Attempt to use the correct method of evaluation	M1	
		Obtain at least two correct simplified terms of the three terms of the expansion of the triple product or of the corresponding determinant, e.g. $-4, -8, -15$	A1	
		Obtain correct non-zero value, e.g. -27, and state that the lines do not intersect	A1	[4]
(ii)		Carry out the correct process for evaluating scalar product of direction vectors for l and m	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 47.1° or 0.822 radians	A1	[3]
(iii)	*EITHER*:	Use scalar product to obtain $a-b+2c=0$	B1	
		Obtain $a+2b-2c=0$, or equivalent, from a scalar product, or by subtracting two point equations obtained from points on m, and solve for one ratio, e.g. $a:b$	M1*	
		Obtain $a:b:c=-2:4:3$, or equivalent	A1	
		Substitute coordinates of a point on m and values for a, b and c in general equation and evaluate d	M1(dep*)	
		Obtain answer $-2x+4y+3z=26$, or equivalent	A1	
	OR1:	Attempt to calculate vector product of direction vectors of l and m	M1*	
		Obtain two correct components	A1	
		Obtain $-2\mathbf{i}+4\mathbf{j}+3\mathbf{k}$, or equivalent	A1	
		Form a plane equation and use coordinates of a relevant point to evaluate d	M1(dep*)	
		Obtain answer $-2x+4y+3z=26$, or equivalent	A1	
	OR2:	Form a two-parameter plane equation using relevant vectors	M1*	
		State a correct equation e.g. $\mathbf{r}=2\mathbf{j}+6\mathbf{k}+s(\mathbf{i}-\mathbf{j}+2\mathbf{k})+t(\mathbf{i}+2\mathbf{j}-2\mathbf{k})$	A1	
		State three correct equations in x, y, z, s and t	A1	
		Eliminate s and t	M1(dep*)	
		Obtain answer $-2x+4y+3z=26$, or equivalent	A1	[5]

Q5.

8	(i)	Either	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line) or equivalent	B1	
			Use scalar product to find cosine of angle between PA and line	M1	
			Obtain $\frac{42}{\sqrt{14\times 230}}$ or equivalent	A1	
			Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 1	Obtain $\pm\begin{pmatrix}2n+2\\n-1\\3n-15\end{pmatrix}$ for PN (where N is foot of perpendicular)	B1	
			Equate scalar product of PN and line direction to zero Or equate derivative of PN^2 to zero Or use Pythagoras' theorem in triangle PNA to form equation in n	M1	
			Solve equation and obtain $n = 3$	A1	
			Obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 2	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line)	B1	
			Evaluate vector product of PA and line direction	M1	
			Obtain $\pm\begin{pmatrix}12\\-36\\-4\end{pmatrix}$	A1	
			Divide modulus of this by modulus of line direction and obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 3	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line)	B1	
			Evaluate scalar product of PA and line direction to obtain distance AN	M1	
			Obtain $3\sqrt{14}$ or equivalent	A1	
			Use Pythagoras' theorem in triangle PNA and obtain $\sqrt{104}$ or 10.2 or equivalent	A1	
		Or 4	Obtain $\pm\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ for vector PA (where A is point on line)	B1	
			Use a second point B on line and use cosine rule in triangle ABP to find angle A or angle B or use vector product to find area of triangle	M1	
			Obtain correct answer (angle A = 42.25…)	A1	
			Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent	A1	[4]

(ii) Either Use scalar product to obtain a relevant equation in a, b, c, e.g. $2a + b + 3c = 0$ or $2a - b - 15c = 0$ M1
State two correct equations in a, b and c A1√
Solve simultaneous equations to obtain one ratio M1
Obtain $a : b : c = -3 : 9 : -1$ or equivalent A1
Obtain equation $-3x + 9y - z = 28$ or equivalent A1

Or 1 Calculate vector product of two of $\begin{pmatrix}2\\1\\3\end{pmatrix}$, $\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ and $\begin{pmatrix}8\\2\\-6\end{pmatrix}$ or equiv M1
Obtain two correct components of the product A1√
Obtain correct $\begin{pmatrix}-3\\9\\-1\end{pmatrix}$ or equivalent A1
Substitute in $-3x + 9y - z = d$ to find d or equivalent M1
Obtain equation $-3x + 9y - z = 28$ or equivalent A1

Or 2 Form a two-parameter equation of the plane M1
Obtain $\mathbf{r} = \begin{pmatrix}1\\3\\-4\end{pmatrix} + s\begin{pmatrix}2\\1\\3\end{pmatrix} + t\begin{pmatrix}2\\-1\\-15\end{pmatrix}$ or equivalent A1√
State three equations in x, y, z, s, t A1
Eliminate s and t M1
Obtain equation $3x - 9y + z = -28$ or equivalent A1 [5]

Q6.

9 (i) Express general point of l or m in component form, i.e. $(3-\lambda, -2+2\lambda, 1+\lambda)$ or $(4+a\mu, 4+b\mu, 2-\mu)$ B1
Equate components and eliminate either λ or μ from a pair of equations M1
Eliminate the other parameter and obtain an equation in a and b M1
Obtain the given answer A1 [4]

(ii) Using the correct process equate the scalar product of the direction vectors to zero M1*
Obtain $-a + 2b - 1 = 0$, or equivalent A1
Solve simultaneous equations for a or for b M1(dep*)
Obtain $a = 3$, $b = 2$ A1 [4]

(iii) Substitute found values in component equations and solve for λ or for μ M1
Obtain answer $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ from either $\lambda = 2$ or from $\mu = -1$ A1 [2]

Q7.

6 (i) State or imply A is $(1, 4, -2)$ B1
State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent B1
Use QP as normal and A as mid-point to find equation of plane M1
Obtain $12x + 6y - 6z = 48$ or equivalent A1 [4]

(ii) Either State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda\mathbf{i}$ B1
Set up and solve a relevant equation for λ. M1
Obtain $\lambda = -9$ and hence B is $(-2, 7, -5)$ A1
Use correct method to find distance between A and B. M1
Obtain 5.20 A1

Or Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent B1
Use correct method involving moduli, scalar product and cosine to find angle APB M1
Obtain 35.26° or equivalent A1
Use relevant trigonometry to find AB M1
Obtain 5.20 A1 [5]

Q8.

10 (i) Equate scalar product of direction vector of l and p to zero M1
Solve for a and obtain $a = -6$ A1 [2]

(ii) Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$ B1
Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ M1
Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a-4) = 0$ or $(1+\mu)(a-4) = 0$ A1
Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied A1 [4]

(iii) Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation M1*
Use $\frac{2}{\sqrt{5}}$ as sine of the angle A1
State equation in any form, e.g. $\frac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \frac{2}{\sqrt{5}}$ A1
Solve for a M1 (dep*)
Obtain answers for $a = 0$ and $a = \frac{60}{31}$, or equivalent A1 [5]
[Allow use of the cosine of the angle to score M1M1.]

Q9.

6	(i) *EITHER*:	State that the position vector of M is $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$, or equivalent	B1	
		Carry out a correct method for finding the position vector of N	M1	
		Obtain answer $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, or equivalent	A1	
		Obtain vector equation of MN in any correct form, e.g. $\mathbf{r}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}-3\mathbf{j}+3\mathbf{k})$	A1	
	OR:	State that the position vector of M is $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$, or equivalent	B1	
		Carry out a correct method for finding a direction vector for MN	M1	
		Obtain answer, e.g. $\mathbf{i}-3\mathbf{j}+3\mathbf{k}$, or equivalent	A1	
		Obtain vector equation of MN in any correct form, e.g. $\mathbf{r}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}-3\mathbf{j}+3\mathbf{k})$	A1	[4]
		[SR: The use of $AN = AC/3$ can earn M1A0, but $AN = AC/2$ gets M0A0.]		
	(ii)	State equation of BC in any correct form, e.g. $\mathbf{r}=3\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\mu(\mathbf{i}-5\mathbf{j}+5\mathbf{k})$	B1	
		Solve for λ or for μ	M1	
		Obtain correct value of λ, or μ, e.g. $\lambda = 3$, or $\mu = 2$	A1	
		Obtain position vector $5\mathbf{i}-8\mathbf{j}+7\mathbf{k}$	A1	[4]

Q10.

7	(i)	State correct equation in any form, e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+2\mathbf{j}-2\mathbf{k})$	B1	[1]
	(ii) *EITHER*:	Equate a relevant scalar product to zero and form an equation in λ	M1	
	OR 1:	Equate derivative of OP^2 (or OP) to zero and form an equation in λ	M1	
	OR 2:	Use Pythagoras in OAP or OBP and form an equation in λ	M1	
		State a correct equation in any form	A1	
		Solve and obtain $\lambda = -\frac{1}{6}$ or equivalent	A1	
		Obtain final answer $\overrightarrow{OP}=\frac{2}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}+\frac{7}{3}\mathbf{k}$, or equivalent	A1	[4]
	(iii) *EITHER*:	State or imply $\overrightarrow{OP}$ is a normal to the required plane	M1	
		State normal vector $2\mathbf{i}+5\mathbf{j}+7\mathbf{k}$, or equivalent	A1√	
		Substitute coordinates of a relevant point in $2x+5y+7z=d$ and evaluate d	M1	
		Obtain answer $2x+5y+7z=26$, or equivalent	A1	
	OR 1:	Find a vector normal to plane AOB and calculate its vector product with a direction vector for the line AB	M1*	
		Obtain answer $2\mathbf{i}+5\mathbf{j}+7\mathbf{k}$, or equivalent	A1	
		Substitute coordinates of a relevant point in $2x+5y+7z=d$ and evaluate d	M1(dep*)	
		Obtain answer $2x+5y+7z=26$, or equivalent	A1	
	OR 2:	Set up and solve simultaneous equations in a, b, c derived from zero scalar products of $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ with **(i)** a direction vector for line AB, **(ii)** a normal to plane OAB	M1*	
		Obtain $a:b:c=2:5:7$, or equivalent	A1	
		Substitute coordinates of a relevant point in $2x+5y+7z=d$ and evaluate d	M1(dep*)	
		Obtain answer $2x+5y+7z=26$, or equivalent	A1	
	OR 3:	With $Q\ (x, y, z)$ on plane, use Pythagoras in OPQ to form an equation in x, y and z	M1*	
		Form a correct equation	A1√	
		Reduce to linear form	M1(dep*)	
		Obtain answer $2x+5y+7z=26$, or equivalent	A1	
	OR 4:	Find a vector normal to plane AOB and form a 2-parameter equation with relevant vectors, e.g., $\mathbf{r}=\mathbf{i}+2\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}-2\mathbf{j}+2\mathbf{k})+\mu(8\mathbf{i}-6\mathbf{j}+2\mathbf{k})$	M1*	
		State three correct equations in x, y, z, λ and μ	A1	
		Eliminate λ and μ	M1(dep*)	
		Obtain answer $2x+5y+7z=26$, or equivalent	A1	[4]

Q11.

6	(i)	State general vector for point on line, e.g. $-5\mathbf{i}+3\mathbf{j}+6\mathbf{k}+s(10\mathbf{i}+5\mathbf{j}-5\mathbf{k})$ or $5\mathbf{i}+8\mathbf{j}+\mathbf{k}+t(10\mathbf{i}+5\mathbf{j}-5\mathbf{k})$ or equiv	B1	
		Substitute their line into equation of plane and solve for parameter	M1	
		Obtain correct value, $s=\frac{2}{5}$ or $t=-\frac{3}{5}$ or equivalent	A1	
		Obtain $(-1, 5, 4)$ o.e.	A1	[4]
	(ii)	State or imply normal vector to p is $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$	B1	
		Carry out process for evaluating scalar product of two relevant vectors	M1	
		Using correct process for moduli, divide scalar product by the product of the moduli and evaluate arcsin(..) or arccos(..) of the result.	M1	
		Obtain 5.1° or 0.089 rads	A1	[4]

Q12.

7	(i)	Use a correct method to express $\overrightarrow{OP}$ in terms of λ	M1	
		Obtain the given answer	A1	[2]
	(ii)	*EITHER*: Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$ in terms of λ	M1	
		Using the correct method for the moduli, divide scalar products by products of moduli and express cos AOP = cos BOP in terms of λ, or in terms of λ and OP	M1*	
		OR1: Use correct method to express $OA^2+OP^2-AP^2$, or $OB^2+OP^2-BP^2$ in terms of λ	M1	
		Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express cos AOP = cos BOP in terms of λ, or λ and OP	M1*	
		Obtain a correct equation in any form, e.g. $\dfrac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}}=\dfrac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$	A1	
		Solve for λ	M1(dep*)	
		Obtain $\lambda=\frac{3}{8}$	A1	[5]
		[SR: The M1* can also be earned by equating cos AOP or cos BOP to a sound attempt at cos $\frac{1}{2}$ AOB and obtaining an equation in λ. The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.]		
		[SR: Allow a solution reaching $\lambda=\frac{3}{8}$ after cancelling identical incorrect expressions for OP to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]		
	(iii)	Verify the given statement correctly	B1	[1]

Q13.

9	(i)	Calculate scalar product of direction of l and normal to p	M1	
		Obtain $4 \times 2 + 3 \times (-2) + (-2) \times 1 = 0$ and conclude accordingly	A1	[2]
	(ii)	Substitute $(a, 1, 4)$ in equation of p and solve for a	M1	
		Obtain $a = 4$	A1	[2]
	(iii)	**Either** Attempt use of formula for perpendicular distance using $(a, 1, 4)$	M1	
		Obtain at least $\frac{2a-2+4-10}{\sqrt{4+4+1}} = 6$	A1	
		Obtain $a = 13$	A1	
		Attempt solution of $\frac{2a-8}{3} = -6$	M1	
		Obtain $a = -5$	A1	
		Or Form equation of parallel plane and substitute $(a, 1, 4)$	M1	
		Obtain $\frac{2a+2}{3} - \frac{10}{3} = 6$	A1	
		Obtain $a = 13$	A1	
		Solve $\frac{2a+2}{3} - \frac{10}{3} = -6$	M1	
		Obtain $a = -5$	A1	
		Or State a vector from a pt on the plane to $(a, 1, 4)$ *e.g.* $\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} a \\ 1 \\ -6 \end{pmatrix}$	B1	
		Calculate the component of this vector in the direction of the unit normal and equate to 6 : $\frac{1}{3}\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}.\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 6$	M1	
		Obtain $a = 13$	A1	
		Solve $\frac{1}{3}\begin{pmatrix} a-5 \\ 1 \\ 4 \end{pmatrix}.\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = -6$	M1	
		Obtain $a = -5$	A1	

	Or		
	State or imply perpendicular line $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$	B1	
	Substitute components for p and solve for μ	M1	
	Obtain $\mu = \dfrac{8-2a}{9}$	A1	
	Equate distance between $(a, 1, 4)$ and foot of perpendicular to ±6	M1	
	Obtain $\dfrac{3(8-2a)}{9} = \pm 6$ or equivalent and hence −5 and 13	A1	[5]

Q14.

10	**(i)**	*EITHER*	Use scalar product of relevant vectors, or subtract point equations to form two equations in a,b,c, e.g. $a - 5b - 3c = 0$ and $a - b - 3c = 0$	M1*
			State two correct equations in a,b,c	A1
			Solve simultaneous equations and find one ratio, e.g. $a : c$, or $b = 0$	M1 (dep*)
			Obtain $a : b : c = 3 : 0 : 1$, or equivalent	A1
			Substitute a relevant point in $3x + z = d$ and evaluate d	M1 (dep*)
			Obtain equation $3x + z = 13$, or equivalent	A1
		OR 1	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} - \mathbf{j} - 3\mathbf{k})$	M2*
			Obtain 2 correct components of the product	A1
			Obtain correct product, e.g. $12\mathbf{i} + 4\mathbf{k}$	A1
			Substitute a relevant point in $12x + 4z = d$ and evaluate d	M1 (dep*)
			Obtain $3x + z = 13$, or equivalent	A1
		OR 2	Attempt to form 2–parameter equation for the plane with relevant vectors	M2*
			State a correct equation e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 3\mathbf{k})$	A1
			State 3 equations in x, y, z, λ and μ	A1
			Eliminate λ and μ	M1 (dep*)
			Obtain equation $3x + z = 13$, or equivalent	A1 [6]
	(ii)	*EITHER*	Find $\overrightarrow{CP}$ for a point P on AB with a parameter t, e.g. $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + t(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	B1 √
			Either: Equate scalar product $\overrightarrow{CP}.\overrightarrow{AB}$ to zero and form an equation in t	
			Or 1: Equate derivative for CP^2 (or CP) to zero and form an equation in t	
			Or 2: Use Pythagoras in triangle CPA (or CPB) and form an equation in t	M1
			Solve and obtain correct value of t, e.g. $t = -2$	A1
			Carry out a complete method for finding the length of CP	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1

OR 1	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 ✓
	Using a relevant scalar product find the cosine of *CAB* (or *CBA)*	M1
	Obtain cost *CAB* = $-\dfrac{22}{\sqrt{11}.\sqrt{62}}$, or cos *CBA* = $\dfrac{33}{\sqrt{11}.\sqrt{117}}$, or equivalent	A1
	Use trig to find the length of the perpendicular	M1
	Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
OR 2	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 ✓
	Using a relevant scalar product find the length of the projection *AC* (or *BC*) on *AB*	M1
	Obtain answer $2\sqrt{11}$ (or), $3\sqrt{11}$ or equivalent	A1
	Use Pythagoras to find the length of the perpendicular	M1
	Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
OR 3	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 ✓
	Calculate their vector product, e.g. $(-2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	M1
	Obtain correct product, e.g. $-2\mathbf{i} + 13\mathbf{j} - 5\mathbf{k}$	A1
	Divide modulus of the product by the modulus of $\overrightarrow{AB}$	M1
	Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
OR 4	State two of $\overrightarrow{AB}$, $\overrightarrow{BC}$) and $\overrightarrow{AC}$ in component form	B1 ✓
	Use cosine formula in triangle *ABC* to find cos *A* or cos *B*	M1
	Obtain cos *A* = $-\dfrac{44}{2\sqrt{11}.\sqrt{62}}$, or cos *B* = $\dfrac{66}{2\sqrt{11}.\sqrt{117}}$	A1
	Use trig to find the length of the perpendicular	M1

Obtain answer $3\sqrt{2}$ (4.24), or equivalent A1 [5]

[The f.t is on $\overrightarrow{AB}$]

Q15.

8	**(i)**	State or imply general point of either line has coordinates $(5+s,\ 1-s,\ -4+3s)$ or $(p+2t,\ 4+5t,\ -2-4t)$		B1	
		Solve simultaneous equations and find s and t		M1	
		Obtain $s=2$ and $t=-1$ or equivalent in terms of p		A1	
		Substitute in third equation to find $p=9$		A1	
		State point of intersection is $(7, -1, 2)$		A1	[5]
	(ii)	<u>Either</u>	Use scalar product to obtain a relevant equation in a, b, c e.g. $a-b+3c=0$ or $2a+5b-4c=0$	M1	
			State two correct equations in a, b, c	A1	
			Solve simultaneous equations to obtain at least one ratio	DM1	
			Obtain $a:b:c=-11:10:7$ or equivalent	A1	
			Obtain equation $-11x+10y+7z=-73$ or equivalent with integer coefficients	A1	
		<u>Or 1</u>	Calculate vector product of $\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and $\begin{pmatrix}2\\5\\-4\end{pmatrix}$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct $\begin{pmatrix}-11\\10\\7\end{pmatrix}$ or equivalent	A1	
			Substitute coordinates of a relevant point in $\mathbf{r}.\mathbf{n}=d$ to find d	DM1	
			Obtain equation $-11x+10y+7z=-73$ or equivalent with integer coefficients	A1	
		<u>Or 2</u>	Using relevant vectors, form correctly a two-parameter equation for the plane	M1	
			Obtain $\mathbf{r}=\begin{pmatrix}5\\1\\-4\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}+\mu\begin{pmatrix}2\\5\\-4\end{pmatrix}$ or equivalent	A1	
			State three equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	DM1	
			Obtain $11x-10y-7z=73$ or equivalent with integer coefficients	A1	[5]

Q16.

9	(i)	*EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
			Use scalar product to obtain an equation in *a, b, c*, e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
			Obtain two correct equations in *a, b, c*	A1	
			Solve to obtain ratio $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR1:	Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
			Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
			Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
			Solve to obtain their ratio, e.g. $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
			Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
			Substitute in $9x + 3y - 6z = d$ to find d	M1	
			Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
		OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
			Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
			Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
			State three correct equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	M1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	[6]
	(ii)		Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	[1]

(iii) *EITHER:* Use $\frac{\overrightarrow{OA}.\overrightarrow{OD}}{\left\|\overrightarrow{OD}\right\|}$ to find projection ON of OA onto OD	M1	
Obtain $ON = \frac{4}{3}$	A1	
Use Pythagoras in triangle OAN to find AN	M1	
Obtain the given answer	A1	
OR1: Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$	M1	
Obtain answer $6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$	A1	
Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$	M1	
Obtain the given answer	A1	
OR2: Taking general point P of OD to have position vector $\lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle OPA, or setting the derivative of $\left\|\overrightarrow{AP}\right\|$ to zero	M1	
Solve and obtain $\lambda = \frac{4}{9}$	A1	
Carry out method to calculate AP when $\lambda = \frac{4}{9}$	M1	
Obtain the given answer	A1	
OR3: Use a relevant scalar product to find the cosine of AOD or ADO	M1	
Obtain $\cos AOD = \frac{4}{9}$ or $\cos ADO = \frac{5}{3\sqrt{10}}$, or equivalent	A1	
Use trig to find the length of the perpendicular	M1	
Obtain the given answer	A1	
OR4: Use cosine formula in triangle AOD to find $\cos AOD$ or $\cos ADO$	M1	
Obtain $\cos AOD = \frac{8}{18}$ or $\cos ADO = \frac{10}{6\sqrt{10}}$, or equivalent	A1	
Use trig to find the length of the perpendicular	M1	
Obtain the given answer	A1	**[4]**

Q17.

6	(i)	Find scalar product of the normals to the planes	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find $\cos^{-1}$ of the result.	M1	
		Obtain 67.8° (or 1.18 radians)	A1	[3]
	(ii)	EITHER Carry out complete method for finding point on line	M1	
		Obtain one such point, e.g. $(2,-3,0)$ or $\left(\frac{17}{7},0,\frac{6}{7}\right)$ or $(0,-17,-4)$ or …	A1…	
		Either State $3a-b+2c=0$ and $a+b-4c=0$ or equivalent	B1	
		Attempt to solve for one ratio, e.g. $a:b$	M1	
		Obtain $a:b:c=1:7:2$ or equivalent	A1	
		State a correct final answer, e.g. r = [2, –3, 0] + λ [1, 7, 2]	A1√	
		Or 1 Obtain a second point on the line	A1	
		Subtract position vectors to obtain direction vector	M1	
		Obtain [1, 7, 2] or equivalent	A1	
		State a correct final answer, e.g. r = [2, –3, 0] + λ [1, 7, 2]	A1√	
		Or 2 Use correct method to calculate vector product of two normals	M1	
		Obtain two correct components	A1	
		Obtain [2, 14, 4] or equivalent	A1	
		State a correct final answer, e.g. r = [2, –3, 0] + λ [1, 7, 2]	A1√	
		[√ is dependent on both M marks in all three cases]		
		OR 3 Express one variable in terms of a second variable	M1	
		Obtain a correct simplified expression, e.g. $x=\frac{1}{2}(4+z)$	A1	
		Express the first variable in terms of third variable	M1	
		Obtain a correct simplified expression, e.g. $x=\frac{1}{7}(17+y)$	A1	
		Form a vector equation for the line	M1	
		State a correct final answer, e.g. r = [0, –17, –4] + λ [1, 7, 2]	A1	
		OR 4 Express one variable in terms of a second variable	M1	
		Obtain a correct simplified expression, e.g. $z=2x-4$	A1	
		Express third variable in terms of the second variable	M1	
		Obtain a correct simplified expression, e.g. $y=7x-17$	A1	
		Form a vector equation for the line	M1	
		State a correct final answer, e.g. r = [0, –17, –4] + λ [1, 7, 2]	A1	[6]

Q18.

7	(i)	Obtain $2x-3y+6z$ for LHS of equation	B1	
		Obtain $2x-3y+6z=23$	B1	[2]
	(ii)	Either Use correct formula to find perpendicular distance	M1	
		Obtain unsimplified value $\frac{\pm 23}{\sqrt{2^2+(-3)^2+6^2}}$, following answer to (i)	A1√	
		Obtain $\frac{23}{7}$ or equivalent	A1	[3]

	OR 1	Use scalar product of (4, –1, 2) and a vector normal to the plane	M1	
		Use unit normal to plane to obtain $\pm\frac{(8+3+12)}{\sqrt{49}}$	A1	
		Obtain $\frac{23}{7}$ or equivalent	A1	[3]
	OR 2	Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M1	
		Obtain $\mu = \frac{23}{49}$ [and $\left(\frac{46}{49}, -\frac{69}{49}, \frac{138}{49}\right)$ as foot of perpendicular]	A1	
		Obtain distance $\frac{23}{7}$ or equivalent	A1	[3]
(iii)	Either	Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once	M1	
		Obtain $\frac{\lvert 23-k \rvert}{7} = 14$ or equivalent	A1	
		Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$	A1	[3]
	OR	Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$	M1	
		Obtain $2x - 3y + 6z = 121$	A1	
		Obtain $2x - 3y + 6z = -75$	A1	[3]

Q19.

10	**(i)**	Express general point of l in component form, e.g. $(1+3\lambda, 2-2\lambda, -1+2\lambda)$	B1	
		Substitute in given equation of p and solve for λ	M1	
		Obtain final answer $-\frac{1}{2}\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or equivalent, from $\lambda=-\frac{1}{2}$	A1	**3**
	(ii)	State or imply a vector normal to the plane, e.g. $2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$	B1	
		Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1	
		Obtain answer 23.2° (or 0.404 radians)	A1	**4**
	(iii)	*EITHER*: State $2a+3b-5c=0$ or $3a-2b+2c=0$	B1	
		Obtain two relevant equations and solve for one ratio, e.g. $a:b$	M1	
		Obtain $a:b:c=4:19:13$, or equivalent	A1	
		Substitute coordinates of a relevant point in $4x+19y+13z=d$, and evaluate d	M1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	
		*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})\times(3\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-4\mathbf{i}-19\mathbf{j}-13\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $4x+19y+13z=d$	M1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	
		*OR*2: Attempt to form a 2-parameter equation with relevant vectors	M1	
		State a correct equation, e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})+\mu(3\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ and μ	A1	
		Eliminate λ and μ	M1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	
		*OR*3: Using a relevant point and relevant direction vectors, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 3 & -5 \\ 3 & -2 & 2 \end{vmatrix}=0$	A1	
		Attempt to expand the determinant	M1	
		Obtain correct values of two cofactors	A1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	**5**

Q20.

10	**(i)**	*EITHER*:	Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i} - 17\mathbf{j} + 4\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	B1
			Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1
			Solve and obtain $\lambda = 3$	A1
			Carry out a complete method for finding the length of AP	M1
			Obtain the given answer 15 correctly	A1
		*OR*1:	Calling $(4, -9, 9)$ B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}$	B1
			Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	M1
			Obtain correct answer, e.g. $-30\mathbf{i} + 6\mathbf{j} + 33\mathbf{k}$	A1
			Divide the modulus of the product by that of the direction vector	M1
			Obtain the given answer correctly	A1
		*OR*2:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1
			Use a scalar product to find the projection of BA (or AB) on l	M1
			Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}}$	A1
			Use Pythagoras to find the perpendicular	M1

	Obtain the given answer correctly	A1
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1
	Use a scalar product to find the cosine of ABP	M1
	Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}.\sqrt{306}}$	A1
	Use trig. to find the perpendicular	M1
	Obtain the given answer correctly	A1
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1
	Find a second point C on l and use the cosine rule in triangle ABC to find the cosine of angle A, B, or C, or use a vector product to find the area of ABC	M1
	Obtain correct answer in any form	A1
	Use trig. or area formula to find the perpendicular	M1
	Obtain the given answer correctly	A1
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ in any form	B1
	Use correct method to express AP^2 (or AP) in terms of λ	M1
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2+(-17+\lambda)^2+(4-2\lambda)^2$	A1
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1
	Obtain the given answer correctly	A1 [5]

(ii) *EITHER*:	Substitute coordinates of a general point of l in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in a and b	M1*
	Obtain a correct equation, e.g. $4a-9b-27+1=0$	A1
	Obtain a second correct equation, e.g. $-2a+b+6=0$	A1
	Solve for a or for b	M1(dep*)
	Obtain $a=2$ and $b=-2$	A1
OR:	Substitute coordinates of a point of l and obtain a correct equation, e.g. $4a-9b=26$	B1
	EITHER: Find a second point on l and obtain an equation in a and b	M1*
	Obtain a correct equation	A1
	OR: Calculate scalar product of a direction vector for l and a vector normal to the plane and equate to zero	M1*
	Obtain a correct equation, e.g. $-2a+b+6=0$	A1
	Solve for a or for b	M1(dep*)
	Obtain $a=2$ and $b=-2$	A1 [5]

Q21.

7 (i)	State at least two of the equations $1+\lambda=a+\mu$, $4=2+2\mu$, $-2+3\lambda=-2+3a\mu$	B1	
	Solve for λ or for μ	M1	
	Obtain $\lambda=a$ (or $\lambda=a+\mu-1$) and $\mu=1$	A1	
	Confirm values satisfy third equation	A1	[4]
(ii)	State or imply point of intersection is $(a+1, 4, 3a-2)$	B1	
	Use correct method for the modulus of the position vector and equate to 9, following their point of intersection	M*1	
	Solve a three-term quadratic equation in a $(a^2-a-6=0)$	DM*1	
	Obtain –2 and 3	A1	[4]